

CS3110: Data Structures & Functional Programming

Lambda Calculus & Map-Reduce

YOU KNOW THIS METAL
RECTANGLE FULL OF
LITTLE LIGHTS?

YEAH.



I SPEND MOST OF MY LIFE
PRESSING BUTTONS TO MAKE
THE PATTERN OF LIGHTS
CHANGE HOWEVER I WANT.

SOUNDS
GOOD.



BUT TODAY, THE PATTERN
OF LIGHTS IS *ALL WRONG!*

OH GOD! TRY
PRESSING MORE
BUTTONS!
IT'S NOT
HELPING!



Logistics

- First Quiz in class Friday
- No class Monday
- I'm behind on creating homeworks and the project. I will try to get the project out tomorrow to give you the weekend.

Homework 1 Solutions

- Hello World – have to reload in order to observe changes when using interactive python
- Everyone did fine.

Homework 2 Solutions

Little more variation on answers.

```
def FunSum(numbers):  
    return int(bool(numbers)) and (numbers[0] +  
                                   FunSum(numbers[1:]))
```

Note:

```
bool([]) == bool (()) == False
```

Style Guide

- Comments!!
- Whitespace!!
- Variable Use!!

```
#!/usr/bin/python
# -*- coding: utf-8 -*-
```

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.collections import EllipseCollection
```

```
def f(t):
    s1 = np.sin(2*np.pi*t); e1 = np.exp(-t)
    temp= np.absolute((s1*e1))+.05
    return temp
```

```
#!/usr/bin/python
# -*- coding: utf-8 -*-
```

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.collections import EllipseCollection
```

```
"""
```

```
f
```

Takes:

t - a float

Returns:

a float = $|\sin(2\pi t) * \exp(t)| + 0.05$

Helper function to Comparison

```
"""
```

```
def f(t):
```

```
    s1 = np.sin(2*np.pi*t)
```

```
    e1 = np.exp(-t)
```

```
    return np.absolute((s1*e1))+.05
```

"""

Displays a Figure of Data Normally distributed around the function $f(t)$

"""

def Comparison():

plt.figure()

t = np.arange(0.0, 5.0, 0.1)

s = f(t)

nse = np.random.normal(0.0, 0.3, t.shape) * s

plt.plot(s+nse, t, 'b^')

plt.hlines(t, [0], s, lw=2)

plt.xlabel('time (s)'); plt.xlim(xmin=0);

plt.title('Comparison of model with data')

plt.show()

plt.plot(s+nse, t, 'b^')

plt.hlines(t, [0], s, lw=2)

plt.xlabel('time (s)')

plt.title('Comparison of model with data')

plt.xlim(xmin=0)

plt.show()

```
def Ellipses ( ):
    fig=plt.figure()
    x = np.arange(10);  y = np.arange(15)
    X, Y = np.meshgrid(x, y)
    XY = np.hstack((X.ravel()[:,np.newaxis], Y.ravel()[:,np.newaxis]))
    ww = X/10.0; hh = Y/15.0; aa = X*9
```

Displays a Figure of Ellipses:

whose shape is controlled by their (x,y) centers

```
def Ellipses():
```

```
    plt.figure()
```

```
    x = np.arange(10)
```

```
    y = np.arange(15)
```

```
    X, Y = np.meshgrid(x, y)
```

```
    XY = np.hstack((X.ravel()[:,np.newaxis], Y.ravel()[:,np.newaxis]))
```

```
    ww = X/10.0
```

```
    hh = Y/15.0
```

```
    aa = X*9
```

```
ax = plt.subplot(1,1,1)
ec = EllipseCollection( ww, hh, aa, units='x',
                        offsets=XY, transOffset=ax.transData)
```

```
ec.set_array((X+Y).ravel())
ax.add_collection(ec)
ax.autoscale_view()
ax.set_xlabel('X'); ax.set_ylabel('y')
cbar = plt.colorbar(ec)
cbar.set_label('X+Y')
plt.show()
```

```
ec = EllipseCollection( ww,
                        hh,
                        aa,
                        units='x',
                        offsets=XY,
                        transOffset=ax.transData)
```

```
ec.set_array((X+Y).ravel())
```

```
ax = plt.subplot(1,1,1)
ax.add_collection(ec)
ax.autoscale_view()
```

```
ax.set_xlabel('X')
ax.set_ylabel('y')
```

```
cbar = plt.colorbar(ec)
cbar.set_label('X+Y')
```

```
plt.show()
```

(Redo) Substitution Evaluation

- In Left to Right manner, evaluate and replace expressions with their bound value.
- Values are terms that need no further simplification to be understood.

Ex. Evaluating :

If cond1:

 fun1

else:

 fun2

Python

1. reduces (cond1) to true or false (also in the left -> right manner as we've seen in the boolean expressions)
2. Evaluates fun1 or fun2 accordingly, recursively in a left->right manner

Bound v. Unbound

$x = 1$

```
def f(x):  
    return x+2
```

$y = 2 * x$

->

$x = 1$

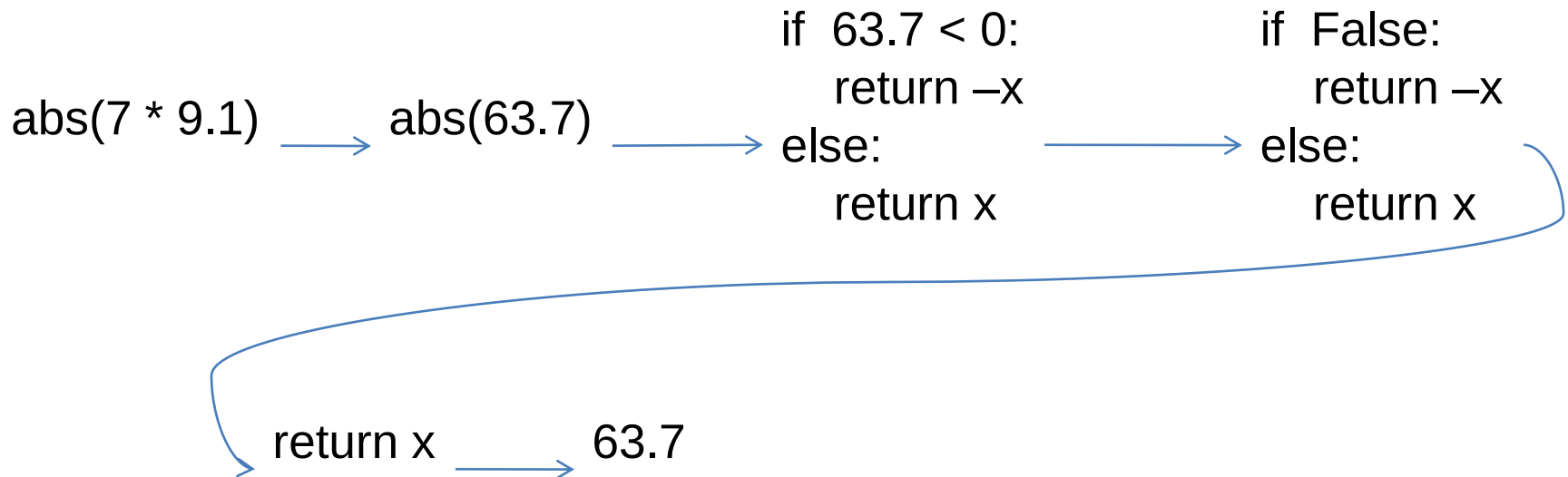
```
def f(x):  
    return x+2
```

$y = 2 * (1) = 2$

Note: x is not Bound with $f(x)$ and hence does not have its value substituted

Simple Substitution Ex.

```
def abs(x):  
    if x < 0:  
        return -x  
    else:  
        return x
```



Normal v Application – order evaluation

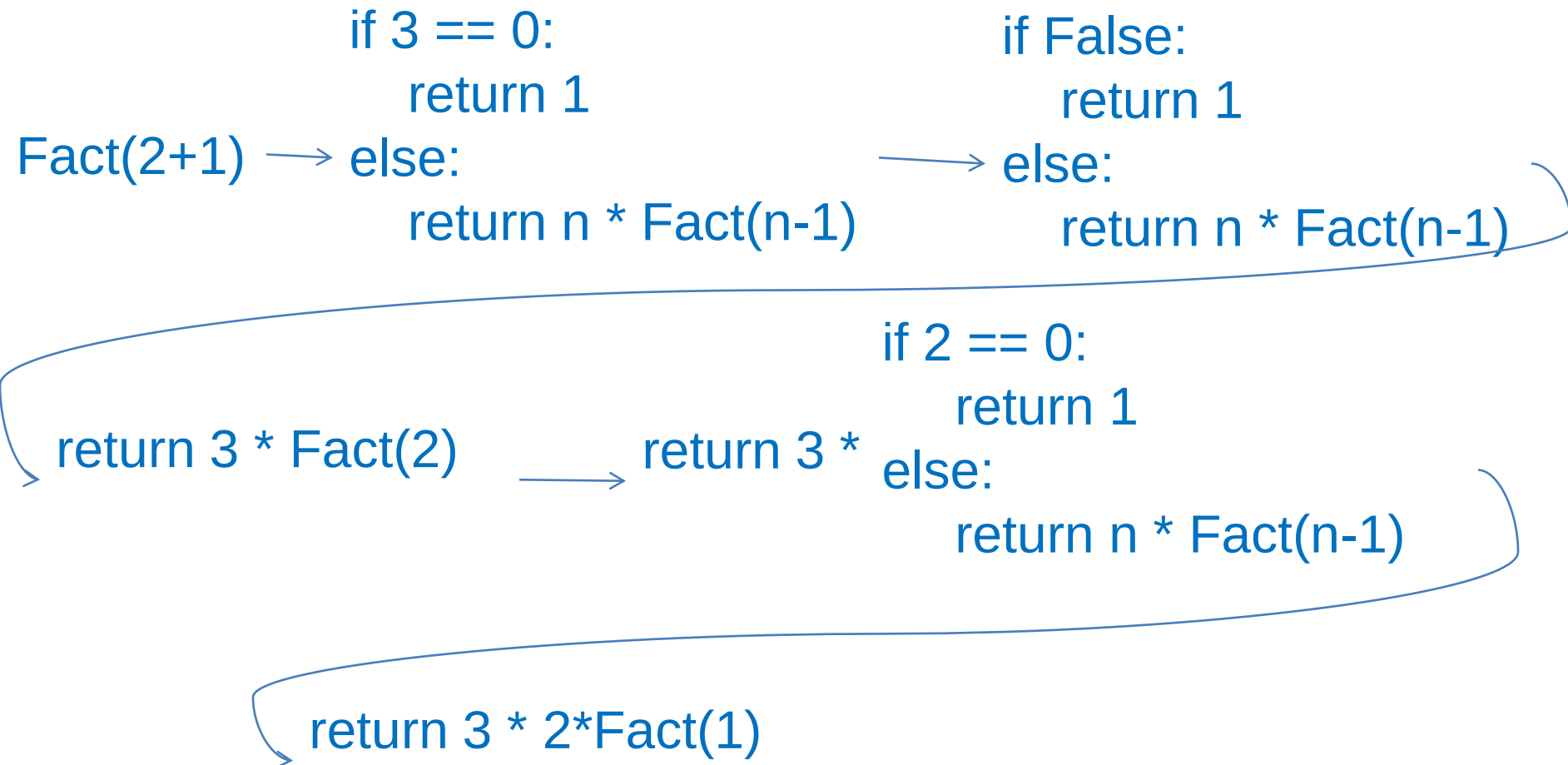
- So far we have been dealing with the Python model of evaluation. Python *eagerly* evaluates, as soon as it can reduce an expression it does. This left-right *eager* evaluation is also known as *applicative-order evaluation*.
- *Normal-order evaluation*: (delayed as long as possible, Haskell, Scala)

$$\begin{aligned}(2+3)^4 &= (2+3) * (2+3) * (2+3) * (2+3) \\ &= (5) * (2+3) * (2+3) * (2+3) \\ &= (5) * (5) * (2+3) * (2+3)\end{aligned}$$

Recursive Substitution

(Application-order evaluation)

```
def Fact(n):  
    if n == 0:  
        return 1  
    else:  
        return n * Fact(n-1)
```



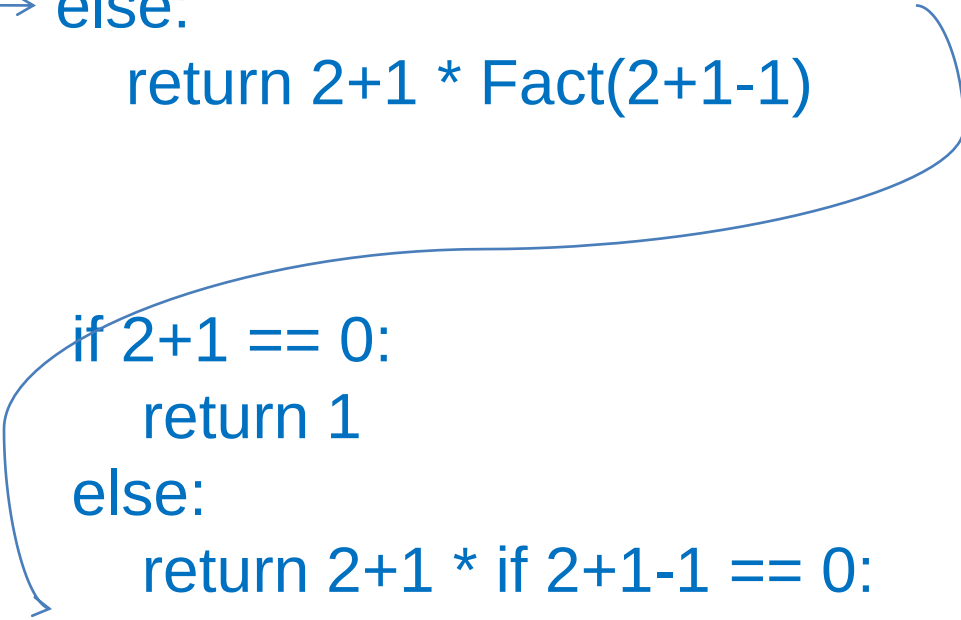
Recursive Substitution

(Normal-order evaluation)

Thinking in the same way quickly gets messy.

```
if 2+1 == 0:  
    return 1
```

Fact(2+1) → else:
 return 2+1 * Fact(2+1-1)



```
if 2+1 == 0:  
    return 1
```

```
else:
```

```
    return 2+1 * if 2+1-1 == 0:  
                  return 1
```

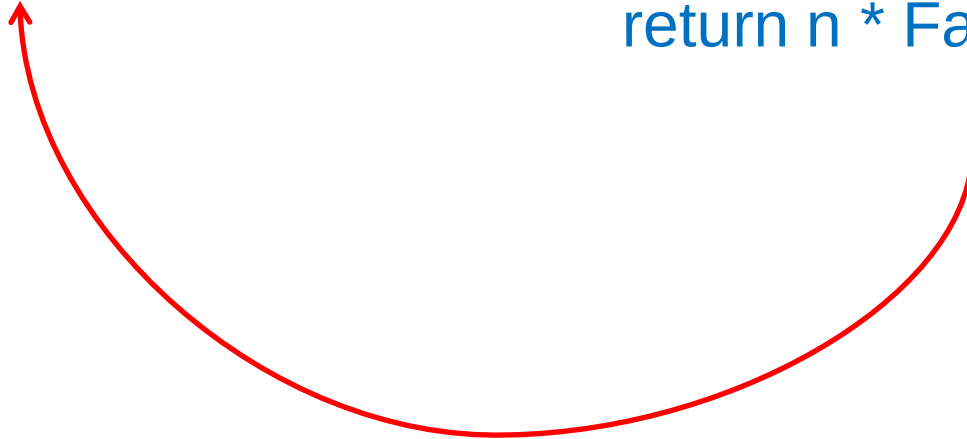
```
    else:
```

```
        return 2+1-1 * Fact(2+1-1-1)
```

Better way of short cutting recursive substitution modeling in Normal-order evaluations. (what's cute here is the idea.)

```
if n == 0:  
    return 1  
else:  
    return n * Fact(n-1)
```

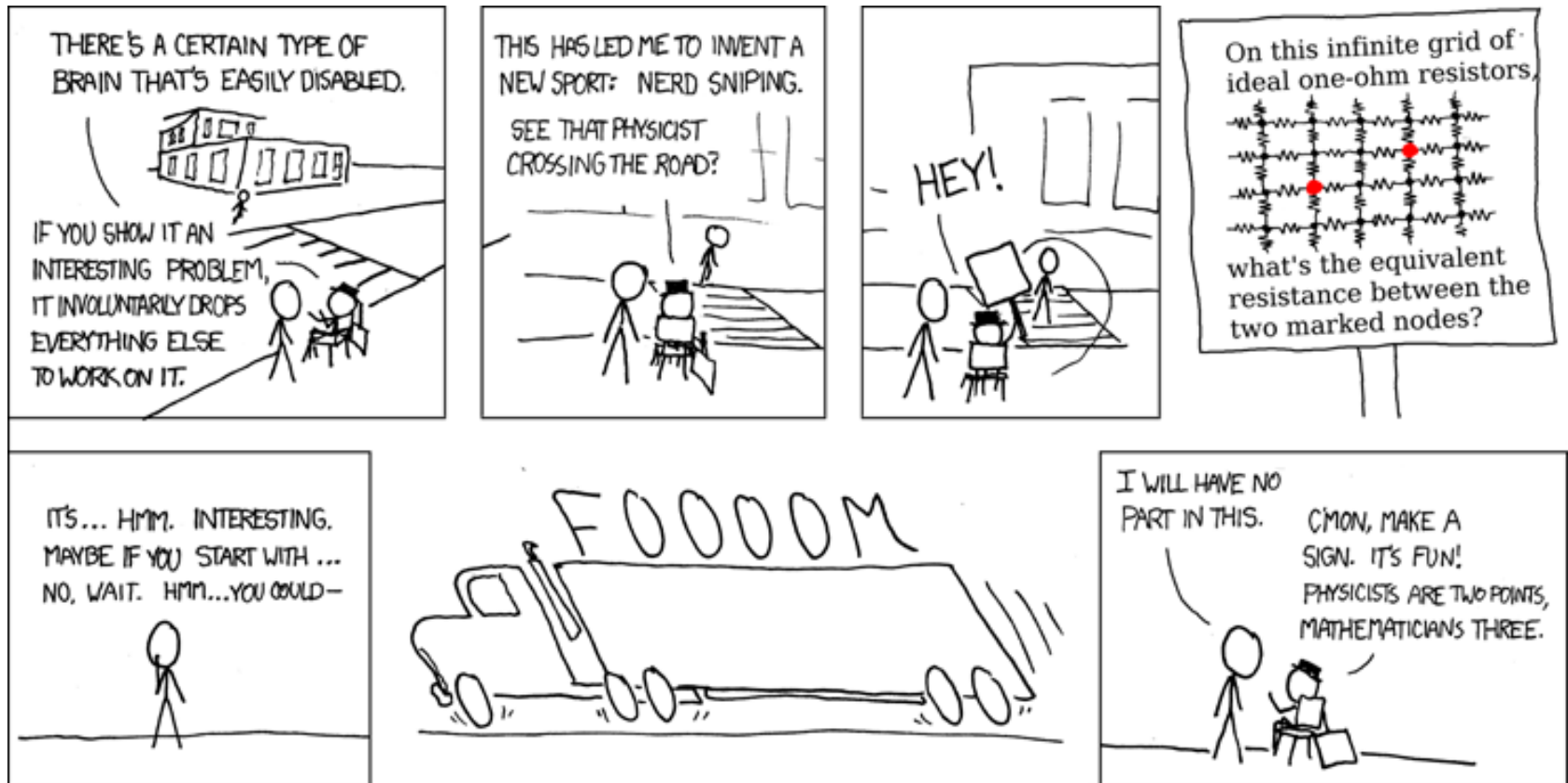
Fact(n)



Where this line represents the recursive substitution.

In programs with no side effects Normal-order and Application-order evaluation models produce the same output. However, as the order of evaluation is different, programs with side effects may produce different outputs.

- Now that we understand evaluation models, let's look at some functional programming staples.



Lambda Function Syntax

From p.78:

- Use a *lambda* function, wherever you would a normal function
- *Lambda* automatically returns an expression
- It's form is:

lambda *parameters: expression*

Ex. Lambda

```
a_list = range(9)
```

```
low = 3
```

```
high = 7
```

```
filtered_list = filter(lambda x, bottom=low, top=high:  
                        top>x>bottom,  
                        a_list)
```

`filter` returns a list of the elements of a list that satisfy the passed function. In this case, `lambda`.

Note: `filter` has no side effects.

- Pros:

1. cute (better organized code)
2. handy to replace once-called functions
(happens many times with calling functions that require another function, filter, sort, etc)

- Cons:

1. in Python can only use 1-line lambdas
2. sometimes harder to read

Clever Use of lambda

- Use lambda to have a function return a function!

```
def SetParameters(para_1, para_2, para_3):  
  
    part_1 = f(para_1, para_2, para_3) # something involved  
  
    return lambda x: x * part_1 + x * g(para_1)
```

- Remember the substitution model?
What does Python do for us here?

Map Function Syntax

From p.164:

- Use a *map* function, wherever you want to iterate a function over an iterable
- No side effects
- Returns a list of the function iterated over the list
- It's form is:

`map(function, list)`

Map or for?

- If you can use map, do. It guarantees no side effects.
- First introduction to thinking about parallel execution. (*board single v. multi-core execution*)
- Because map has no side effects, guarantees each operation can be done independently.
- Google reports using map-reduce for 130,000 machine-months of processing time.

Map Function Syntax

```
capped = map(lambda x: min(0, x), a_list)
```

Returns a list and does not modify `a_list`.