CS 2802: Homework 8

April 12, 2020

Handed out April 13, due Tuesday, April 21 at 5 PM. (No possibility of handing it in late, though.) Note that this is a somewhat long assignment, which is why I'm giving as much time as I can. The reason that it's long is that includes problems on all the probability material that will be covered in the prelim. I plan to post the solutions to the homework on April 21 (after the homework has been handed in, of course!), so you can use them to help in studying for the prelim.

- Read Chapters 18 and 19, except for 19.4.6 and 19.5.4-19.5.8. (The material not covered is still interesting. We just don't the time time to do everything!)
- Do the following problems:
  - 18.7 (make sure that your answer explains Sauron's mistake as well as calculating the correct probability)
  - 18.10 (for (b), state clearly which event we want the conditional probability of)
  - 18.12
  - 18.17
  - 18.24 (Give a concrete counterexample to the step that you think is incorrect.)
  - 18.25(a),(b). (Again, make sure you clearly specify the events that you are taking the probability of.)
  - 18.31(b)
  - 19.1
  - 19.9. (For full credit, you must clearly specify a sample space and random variable on this space that you're taking the expectation of in part (c), use this random variable to define the probabilities of interest in parts (a) and (b).)
  - 19.11

- 19.21 (Again, make clear what the sample space is, and what random variables you're using. Hint: There's a really elegant approach that gives a 5-line solution.)
- 19.22 (Hint: the sample space should consist of all the possible truth assignments to the primitive propositions that appear in G.)
- Extra problem: In the second-ace puzzle, come up with a protocol for Alice that is consistent with the story told in class. Specifically, if Alice says"I have an ace" at the first step, the probability (according to Bob) that she has both aces is 1/5; if she says "I have the ace of spades" at the second step, the probability that she has both aces is 1/3; and if she says "I have the ace of hearts" at the second step, the probability that she has both aces is also 1/3. How does your protocol address the concern that Bob can say (after the first step, when Alice says she has an ace) "no matter Alice tells me she has at the second step, I'll think that she has both aces with probability 1/3, so I should ascribe probability 1/3 to her having both aces now (after the first step)." (Hint: the protocol is quite simple. The easiest way to come up with it is perhaps to think about what you'd want to the probability tree to look like, and then construct a protocol with that probability tree.)
- Challenge problem (you don't have to hand this in): Basketball fans are sad that the NCAA basketball tournament (otherwise known as "March Madness") was canceled this year due to the coronavirus pandemic. So, you'll have to make do with this puzzle: Suppose that there are 64 teams in the tournament. It's a *single-elimination tournament*: as soon as a team loses, it's out. So it goes on for 6 rounds and there are 63 games. (In general, an easy induction shows that a single-elimination tournament with 2^n teams goes on for $n-1$ rounds and there are 2^n-1 games, since each game knocks out one team.) You predict the winners for each of the 63 games. Your score is then computed as follows: 32 points for correctly predicting the final winner, 16 points for each correct finalist, and so on, down to 1 point for every correctly predicted winner for the first round. (The maximum number of points, if you predict all games correctly, you can get is thus 192.) Knowing nothing about any team, you flip fair coins to decide every one of your 63 bets What is your expected score?

Think about (but don't hand in) 18.11, 18.16.