CS 2802: Homework 7

April 4, 2020

Handed out April 6, due April 13

- Read Chapter 17, 18.1-18.4
- Do the following problems:
  - 17.2
  - 17.4. Also compute the probability that each player wins. You should assume that $0 < p < 1$ (whre p is the probability of heads.) (Extra hint: first compute the probability that the first player wins.)
  - 17.6(b),(c),(d) (Parts (c) requires a little calculus, which I assume that you've all had. You can assume that $\log(1-x) \approx -x$, which follows from the Taylor series for log.) For (d), remember that $a^x = e^{\log_e(a)x}$ and that $\log(1+x) = 1 + x + x^2/2 + \cdots$.)
  - 17.9 (We did the Difference Rule and the Inclusion-Exclusion Rule in class; you should just do the other three. You can use the two that we already did in your proof.)
  - 18.2 (you don't need to explain your answer for (a) and (b), but you do need some explanation for part (c)).
  - 18.5. (You don't have to use the four-step method or give a tree diagram, but you do have to give a careful explanation whatever you do.)
- Extra problem: Alice, Bob, and Charlie are all pretty good chess players, but Alice is a little better than Bob. Charlie has to play three games against Alice and Bob. He either plays Alice, Bob, and then Alice, or Bob, Alice, and then Bob. He wants to maximize his chance of winning two games in a row. In which order should he play Alice and Bob. (Of course, you need to prove your answer.) [Formally, assume that the probability that Charlie beats Alice is p_A and the probaiblity that Charlie beats Bob is p_B, where $p_B > pA$. You can assume that these probabilities are the same each time that Charlie plays Bob or Alice.]

- Challenge problem (you don't have to hand this in): Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion. (For example, for 2, $2 \times 5 = 10$; for 3, $3 \times 37 = 111$; for 4, $4 \times 25 = 100$. (There's a short cute answer, but it's not so easy to find. Hint: use the pigeonhole principle and a little bit of number theory.)