CS 2802: Homework 6

March 3, 2020

Handed out March 5, due March 17 (in 12 days)

- Read Chapter 15
- Do the following problems:
  - 15.5(b)
  - 15.6
  - 15.9(a),(b), (c). (You don't have to do the second part of (c), where it asks about the fraction of functions vs. total functions.)
  - 15.12
  - 15.28
  - 15.35(a) (Hint: First think about the number of ways of doing this if the x_is just have to be non-negative, rather than being positive.)
  - 15.37
  - 15.38 (b), (c)
  - 15.56
  - 15.64 (explain how you go your answer in part (b)–(e))
  - 15.65 (just give a combinatorial argument; no need to do the calculations)
  - 15.68(a)
  - Challege problem (no need to hand this in). This is a variant of the muddy children puzzle discussed in class, but harder: A countably infinite number of prisoners, each with an unknown and randomly assigned red or blue hat line up single file line. Each prisoner faces away from the beginning of the line, and each prisoner can see all (infinitely many) hats in front of him, and none of the (finitely many) hats behind. Starting from the beginning of the line, each prisoner must correctly identify the color of his hat or he is killed on the spot. The prisoners have a chance to meet beforehand and discuss

a strategy. They can have unbounded memory, so they can even remember uncountably many things if necessary.) Each prisoner can hear what the other prisoners before him say. Show that there is a strategy that the prisoners can use such that the first prisoner is killed with probability .5, and the remaining prisoners all survive. (As I said, this is tricky. Here's a hint: you can define an equivalence relation on infinite sequences of 0s and 1s such that two sequences are equivalent iff they agree on all but finitely many elements. Given two equivalent sequences, you can consider the parity of the number of elements on which they disagree. Note that if they are in the same equivalence class, they will disagree on only finitely many elements. For example, if they disagree on three places, the parity of the number of places they disagree is 3.]