

# CS 2802: Homework 4

February 9, 2020

Handed out Feb. 10; due Feb. 17

- Read Chapter 5
- Do the following problems:
  - 5.5
  - 5.6(a)
  - 5.7
  - 5.16(a), (c), (d), (e), (h)
  - 5.21
  - 5.24
  - 5.30

and the following:

- Extra Problem 1: Recall the inductive definition of transitive closure of  $R$  given in class:
  - \* Suppose that  $R$  is a relation on  $S$ . Let  $R_0 = R$ .
  - \* Let  $R_{n+1} = R_n \cup \{(s, t) : \exists u \in S((s, u) \in R_n, ((u, t) \in R_n)\}$ .
  - \* Let  $R' = \bigcup_{n=0}^{\infty} R_n$ .

Prove that  $R'$  is the transitive closure of  $R$ .

NOTE: When you do these problems, make clear what  $P(n)$  is, what the base case is, and what the inductive step is. We will take off points for bad presentation!

Here's a challenge problem (not to be handed in), from last year's prelim: Imagine a Grand Hotel with a countably infinitely of rooms, numbered  $1, 2, 3, \dots$ . The hotel is full. When you arrive at the hotel late in the evening, the manager says that there is no problem accommodating you, although the hotel is full. You can go in room 1, and the previous occupant of room  $n$  moves to room  $n + 1$ .

- (a) Now a bus with a countable infinity of passengers arrives. Show how they can be accommodated (you have to give an explicit room assignment).
- (b) Now countably infinitely many buses arrive, each with a countable infinity of passengers. Show explicitly how they can be accommodated, making use of the fact that there are infinitely many primes. Explain carefully why different passengers are in different rooms.
- (c) Could you accommodate everyone if uncountably many cars arrived, each containing only the driver? Explain carefully why or why not. Would it have helped if the drivers carpooled? (Again, explain why or why not.)