Discrete Structures CS2800	Prelim 2

Instructions: This is a 90 minute test. Answer the following questions in the provided booklet. Ensure that your name and netid are on your exam booklet. You may answer the questions in any order, but please mark that questions clearly. Books, notes, calculators, laptops, and carrier pigeons are all disallowed. You may leave mathematical expressions unevaluated (e.g. just write $17 \cdot 3$ instead of 51 and don't bother evaluating $C(17, 3)$). Good luck! **For credit you must show how you arrived at your answer.**

1. **(3 points)** How many solutions (x_1, x_2, x_3, x_4) are there to the equation $x_1 + x_2 + x_3 + x_4 = 35$, where x_1, x_2, x_3, x_4 are natural numbers? (Just write down the combinatorial expression that describes the answer; there's no need to calculate it numerically. Also, recall that 0 is a natural number.)
2. **(3 points)** How many integers between 1 and 100 (inclusive) are divisible by either 3 or 4?
3. **(5 points)** Let S be a sample space. Give a formal mathematical definition of the following:
   (a) Probability measure on S
   (b) Event of S
   (c) Random variable on S
   (d) Independent random variables of S
   (e) $\Pr(A \mid B)$, where A and B are events.
4. **(4 points)** A group of 10 women and 9 men are in a room. If 3 of the 19 are selected at random, what is the probability that all 3 are of the same sex? (Again, just write down the combinatorial expression that describes the answer; there's no need to calculate it numerically.)
5. **(3 points)** Roughly 1% of Cornell's undergraduate students take CS 2800. 95% of the CS 2800 students know the correct definition of "injective", while only 20% of students who didn't take 2800 know the definition. While walking down the hall, you overhear one undergraduate student saying to another "...since f is injective, we know that if $x \neq y$ then $f(x) \neq f(y)$, so ...". What is the probability that the student has taken CS 2800? (Assume that you heard a randomly chosen Cornell undergrad.)
6. **(8 points)** You toss a blue coin that lands heads 1/2 of the time, and a red coin that lands heads 1/3 of the time. You toss the blue coin 5 times and the red coin 3 times. Suppose you get \$2 every time the blue coin lands heads, and you lose \$1 every time the red coin lands heads. (You get nothing if either coin lands tails.)
   (a) Write down a sample space that describes this situation. How many elements does it have?
   (b) Let E be the event that the third toss of the red coin lands tails. What is $\Pr(E)$?
   (c) Let F be the event that the red coin lands heads at least twice. What is $\Pr(F)$?
   (d) Are E and F independent?
   (e) Define a random variable that describes how much money you win. Find its expected value.
7. **(4 points)** Suppose X is the constant random variable c (that is $X(s) = c$ for all s in the sample space). Show that (a) $E(X) = c$ and (b) $\mathrm{Var}(X) = 0$.
8. **(3 points)** The average height of an adult American is about 5.5 feet, and the standard deviation is about 0.2 feet. You wish to build a door that guarantees that 90% of American adults can enter without ducking. Using Chebychev's inequality, how tall must the door be? (*Hint:* If $h \leq E(H) + d$ then $|h - E(H)| \leq d$. Thus $\Pr(H \geq E(H) + d) \leq \Pr(|H - E(H)| \geq d)$).
9. **(3 points)** Is there an undirected graph with 5 vertices all of which have degree 3? (If you think there is such a graph, draw it. If not, explain why not. Just a "yes" or "no" gets no credit.)
10. **(3 points)** Does the graph to the right have a Eulerian path? Does it have an Eulerian cycle? (If you think the answer is yes, say what it is. If not, explain why not.)