Probability 101

CS 2800: Discrete Structures, Spring 2015

Sid Chaudhuri

Thought for the Day #0

What are the chances Cornell will declare a snow day?


New York Zoological Society

Thought for the Day #1

After how many years will a monkey produce the Complete Works of Shakespeare with more than 50% probability?

Thought for the Day #1

After how many years will a monkey produce the Complete Works of Shakespeare with more than 50% probability?

(or just an intelligible tweet?)

Elements of Probability Theory

- Outcome
- Sample Space
- Event
- Probability Space


What's the most you ever lost on a coin toss?
Miramax

Heads

Tails

Outcomes

Heads

Tails

Sample Space

Heads

Tails

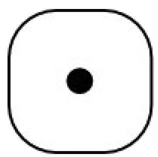
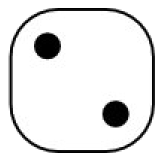
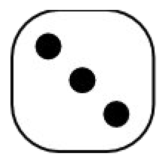
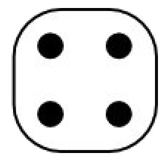
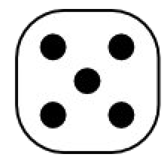
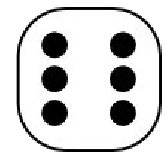
playingintheworldgame.wordpress.com

Sample Space

Set of all possible outcomes of an experiment

Some Sample Spaces

- Coin toss: {  ,  }

- Die roll: {  ,  ,  ,  ,  ,  }

- Weather: {  ,  ,  ,  }

Sample Space

Set of all **mutually exclusive** possible outcomes of an experiment

Event

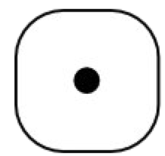
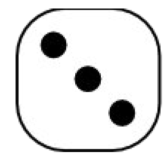
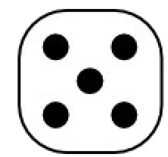
Subset of sample space

Some Events

- Event of a coin landing heads: {  }

Some Events

- Event of a coin landing heads: { 

}
- Event of an odd die roll: { , ,  }

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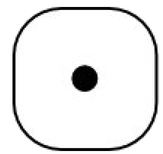
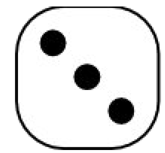
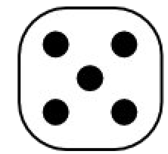
- Event of an odd die roll: {  ,  ,  }
- Event of weather like Ithaca:

{  ,  ,  ,  }

Some Events

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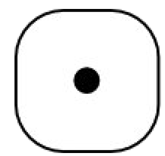
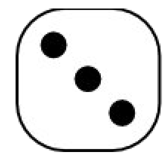
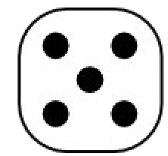
- Event of an odd die roll: {  ,  ,  }
- Event of weather like Ithaca:

  {  ,  ,  ,  }
- Event of weather like California: {  }

Careful!

- The sample space is a **set** (of outcomes)
- An outcome is an **element** of a sample space
- An event is a **set** (a *subset* of the sample space)
  - It can be **empty** (the null event $\{\,\}$ or $\varnothing$, which never happens)
  - It can contain a **single** outcome (simple/elementary event)
  - It can be the **entire** sample space (certain to happen)
- Strictly speaking, an outcome is not an event (it's not even an elementary event)

Probability Space

Sample space S

... plus function P assigning real-valued **probabilities** $P(E)$ to events $E \subseteq S$

... satisfying **Kolmogorov's axioms**

All three are needed!

Kolmogorov's Axioms

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A countable set of events can be indexed by the natural numbers as "first event", "second event", "third event" and so on. We'll see a formal definition of countability later, but for now you don't need to worry about this too much.

2. $P(S) = 1$

3. If a *countable* set of events $E_1, E_2, E_3, \ldots$ are pairwise disjoint ("mutually exclusive"), then

$$P(E_1 \cup E_2 \cup E_3 \cup \ldots) = P(E_1) + P(E_2) + P(E_3) + \ldots$$

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Can you prove these from the axioms?

- In a valid probability space (S, P)
  - $P(E') = 1 - P(E)$ for any event E

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Space for boardwork

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  - $P(E') = 1 - P(E)$ for any event E
  - $P(\emptyset) = 0$

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Space for boardwork

Equiprobable Probability Space

- All outcomes equally likely (fair coin, fair die...)
- Laplace's definition of probability (*only* in finite equiprobable space!)

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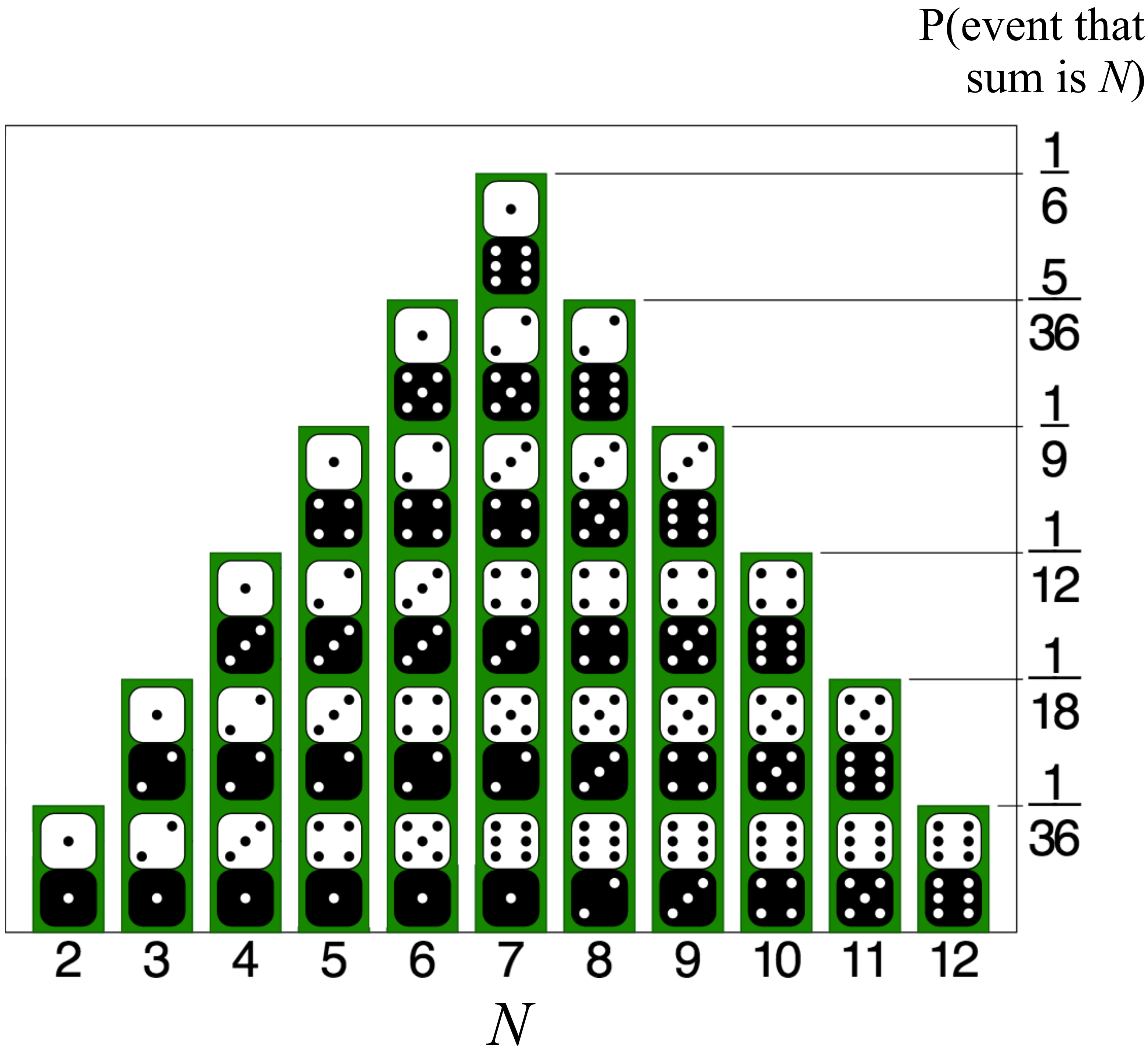
Number of elements (outcomes) in E

Number of elements (outcomes) in S



Tim Stellmach, Wikipedia

Gerolamo Cardano
(1501-1576)

Liar, gambler, lecher, heretic

saderpo@glogster