

1. (a) Suppose that (S, P) is a probability space, and let A be an event of S with $P(A) \neq 0$. Define a function Q by $Q(B) = P(B|A)$. Prove that (A, Q) is a probability space.
(b) Suppose we define R by $R(B) = P(A|B)$. Give a counterexample to show that (A, R) is not a probability space.
2. (a) Give a probability space and a pair of events A and B such that A and B are mutually independent but are not mutually exclusive.
(b) Similarly, give an A and B that mutually exclusive but not mutually independent.
(c) Give an A and B that are neither mutually exclusive nor mutually independent.
(d) Give an A and B that are both mutually exclusive and mutually independent.
3. Show that for finite sample spaces, giving the probability of all of the elementary events is sufficient to determine the probabilities of all events. That is, if S is finite, and if (S, P_1) is a probability space, and if (S, P_2) is a probability space, and if for all $x \in S$, $P_1(\{x\}) = P_2(\{x\})$, then for all $A \subseteq S$, $P_1(A) = P_2(A)$.
This is not true for infinite sample spaces; be sure to indicate in your proof where you used the fact that S is finite.
4. Sid and Mike go to the ice cream store. Sid likes chocolate ice cream, but he doesn't want to order exactly the same flavour that Mike orders. If Mike does not order chocolate ice cream, Sid will order it 90% of the time. If Mike does order chocolate ice cream, Sid will order it only 30% of the time. Mike chooses first. There are 5 different flavours of ice cream (only one is chocolate), and Mike chooses a flavour completely at random (i.e. equiprobably). If Sid ends up buying chocolate ice cream, what is the probability Mike also ordered chocolate ice cream?