## Lecture 4

Lemma:  $|A| \le |B|$  and  $|B| \le |A|$  implies |A| = |B|. some countable sets

finite length strings computer programs all finite cardinality sets of integers all rationals

## Larger sets

$$f: \{1, 2, 3, \ldots\} \rightarrow \{0, 1\}$$
 all sets of integers

Diagonalization over all computable functions

Halting problem not computable

There exists a theorem that is true but has no proof

Between every pairs of rationals there is a real and between every pair of reals there is a rational.