Discrete Structures	Prelim 1
CS2800	March 9, 2015

Instructions: This is a 50 minute exam. Please answer the following questions in the provided booklet. Ensure that your name and netid are on your exam booklet. Clearly indicate your answer to each question. Books, notes, calculators, laptops, and carrier pigeons are all disallowed. You may leave mathematical expressions unevaluated (e.g. just write $17 \cdot 3$ instead of 51). You may use any result proved in class or in the homeworks without proof (ask us if you are unsure).

This exam has **6 (SIX) QUESTIONS** and **2 (TWO) PAGES**. Please turn the sheet over!

1. Let $S = \{a, b\}$, and the function P given by the following table:

A	$P(A)$
$\emptyset$	0
$\{a\}$	1/4
$\{b\}$	3/4
$\{a, b\}$	1

   (a) **(1 point)** What are the events of S?

   (b) **(3 points)** Prove that (S, P) is a probability space.

   (c) **(1 point)** Are $\{a\}$ and $\{b\}$ independent? Explain.

2. **(3 points)** Let E be a herd of 100 elephants. The herd contains 10 adult males, 60 adult females and 30 babies. It is known[1] that the adult elephants have an average surface area of 17m^2, and the babies have an average surface area of 4m^2. A biologist, unaware of these statistics, picks an elephant uniformly at random from E and measures its surface area (after temporarily and painlessly tranquilizing it). If the measured surface area is represented as a random variable, what are its (a) domain, (b) codomain, and (c) expectation (show your calculations)?

   (*Note:* There are many correct answers for (a) and (b). Pick any one.)

3. **(3 points)** For any function $f : A \to B$ and a set $C \subseteq A$, define $f(C) = \{f(x) \mid x \in C\}$. That is, $f(C)$ is the set of images of elements of C. Prove that if f is injective, then $f(C_1 \cap C_2) = f(C_1) \cap f(C_2)$ for all $C_1, C_2 \subseteq A$.

   (*Hint:* one way to prove this is from the definition of set equality: $A = B$ iff $A \subseteq B$ and $B \subseteq A$.)

4. **(3 points)** Suppose R is an equivalence relation on set A. Define the relation R' as the complement of R, that is,

$$R' \;=\; (A \times A) \setminus R \;=\; \{(a, b) \mid a, b \in A \text{ and } (a, b) \notin R\}$$

   Or in other words, for $a, b \in A$,

$$a \; R' \; b \;\text{ if and only if }\; a \text{ is } \textit{not} \text{ related to } b \text{ by } R$$

   Prove or disprove:

   (a) R' is reflexive

   (b) R' is symmetric

   (c) R' is transitive

[1] K. P. Sreekumar and G. Nirmalan, "Estimation of the total surface area in Indian elephants (Elephas maximus indicus)", Veterinary Research Communications, 1990;14(1):5-17.

5. **(2 points)** In an infamous criminal case, a mother was accused of murdering her two infant sons. A well-known statistician testified that the chance that *both* deaths were natural was infinitesimal. He proposed the following calculation:

$$P(D_1 \cap D_2 \mid I) = P(D_1 \mid I) \cdot P(D_2 \mid I)$$

where D_1 and D_2 are the events that the two children respectively died, and I is the event that the mother is innocent.

Since natural infant death is rare in the family's demographic, both probabilities on the right hand side are tiny: about 1/8543. Plugging in the values, we obtain $P(D_1 \cap D_2 \mid I) \approx 1/73{,}000{,}000$. Based on this, the mother was found guilty and imprisoned.

Four years later, the ruling was overturned on grounds of faulty statistics. There are **two** significant errors in the reasoning above. Briefly and clearly identify both.

6. **(0 points)** How would you (humanely) measure the surface area of an elephant?