Discrete Structures CS2800 Fall 2014	Prelim 1

Instructions: This is a 50 minute exam. Please answer the following questions in the provided booklet. Ensure that your name and netid are on your exam booklet. Clearly indicate your answer to each question.

Books, notes, calculators, laptops, and carrier pigeons are all disallowed. You may leave mathematical expressions unevaluated (e.g. just write $17 \cdot 3$ instead of 51).

1. Let $(\mathcal{S}, \mathcal{P})$ be a probability space. Prove that $\mathcal{P}(\emptyset) = 0$. You may use any results from set theory without proof.

2. Let $(\mathcal{S}, \mathcal{P})$ be a probability space, and let A and B be events of $\mathcal{S}$.
   (a) Give the definition of "A and B are independent".
   (b) Give the definition of $\mathcal{P}(A \mid B)$.
   (c) Assume $\mathcal{P}(B) \neq 0$. Prove that if A and B are independent then $\mathcal{P}(A \mid B) = \mathcal{P}(A)$.

3. Sid and Mike go to the ice cream store. Sid likes chocolate ice cream, but he doesn't want to order exactly the same flavour that Mike orders. If Mike does not order chocolate ice cream, Sid will order it 90% of the time. If Mike does order chocolate ice cream, Sid will order it only 30% of the time. Mike chooses first. There are 5 different flavours of ice cream (only one is chocolate), and Mike chooses a flavour completely at random (i.e. equiprobably). If Sid ends up buying chocolate ice cream, what is the probability Mike also ordered chocolate ice cream?

4. Which of these is the correct negation of $\exists x, \neg \forall y, \neg \exists z, \neg F(x, y, z)$?
   (a) $\exists x, \exists y, \exists z, F(x, y, z)$
   (b) $\exists x, \exists y, \exists z, \neg F(x, y, z)$
   (c) $\forall x, \forall y, \forall z, F(x, y, z)$
   (d) $\forall x, \forall y, \forall z, \neg F(x, y, z)$

5. (a) Write the definition of "$f : A \to B$ is injective" using formal notation ($\forall$, $\exists$, $\wedge$, $\vee$, $\neg$, $\Rightarrow$, $=$, $\neq$, $\ldots$).
   (b) Similarly, write down the definition of "$f : A \to B$ is surjective".
   (c) Write down the definition of "A is countable". You may write "f is surjective" or "f is injective" in your expression. (Note: we gave two slightly different definitions of countable in lecture; we will accept either answer).

6. Recall that the composition of two functions $f : B \to C$ and $g : A \to B$ is the function $f \circ g : A \to C$ defined as $(f \circ g)(x) = f(g(x))$. Prove that if f and g are both injective, then $f \circ g$ is injective.

7. For each of the following functions, indicate whether the function f is injective, whether it is surjective, and whether it is bijective. Give a one sentence explanation for each answer.
   (a) $f : \mathbb{N} \to \mathbb{N}$ given by $f : x \to x^2$
   (b) $f : \mathbb{R} \to \mathbb{R}$ given by $f : x \to x^2$
   (c) $f : X \to [Y \to X]$ given by $f : x \mapsto h_x$ where $h_x : Y \to X$ is given by $h_x : y \mapsto x$.