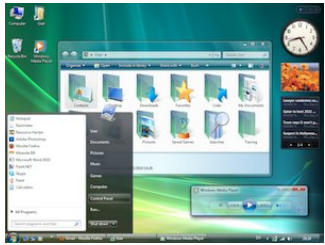


# Finite Automata

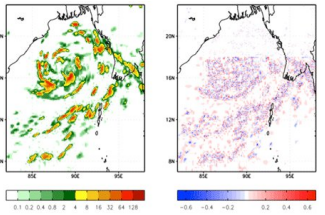
CS 2800: Discrete Structures, Fall 2014

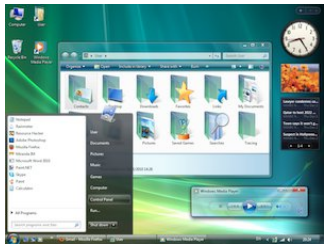
Sid Chaudhuri



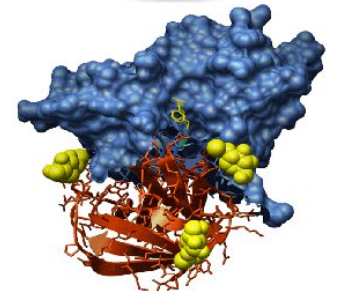
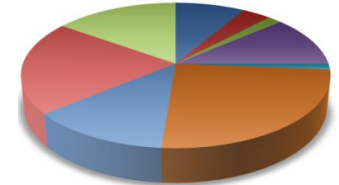
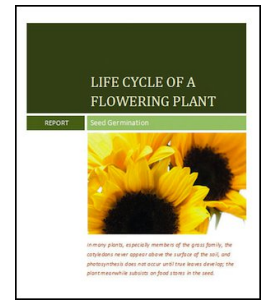
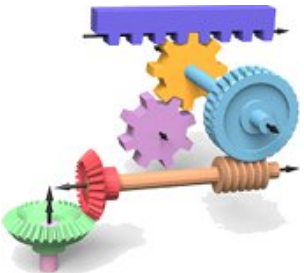
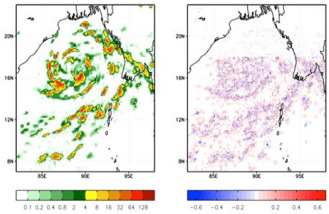


```
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<html id="home-Layout">  
<head>  
<meta http-equiv="content-type" content="text/html; charset=utf-8" />  
<title>Source Code Pro</title>  
<!-- made with <3 and AFDKO -->  
<meta name="keywords" content="sans, monospace, open source, coding, font" />  
<link rel="stylesheet" type="text/css" href="css/main.css" />  
</head>  
<body>  
<div id="main">
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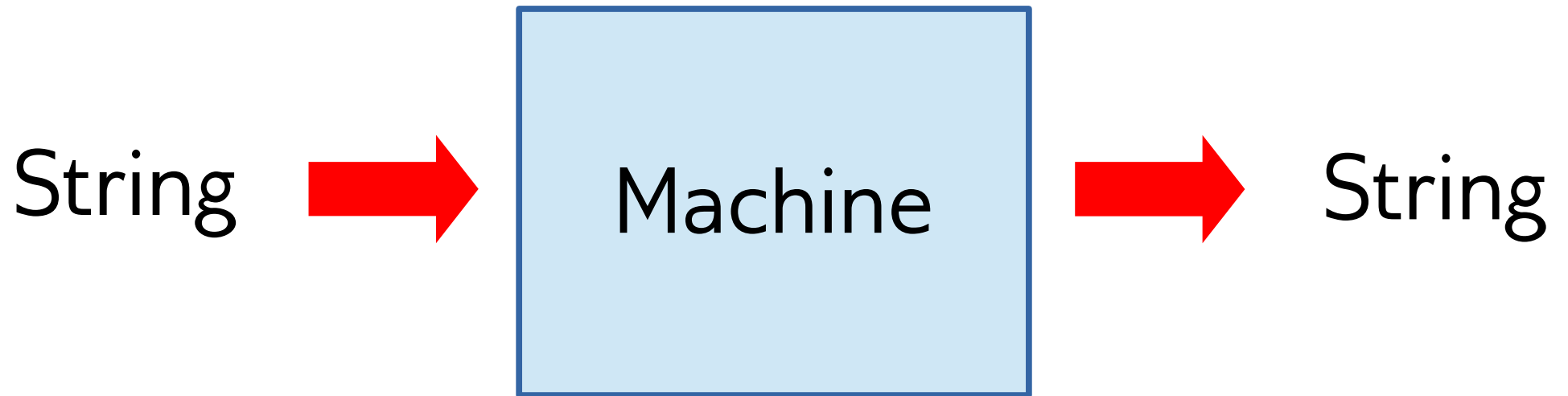




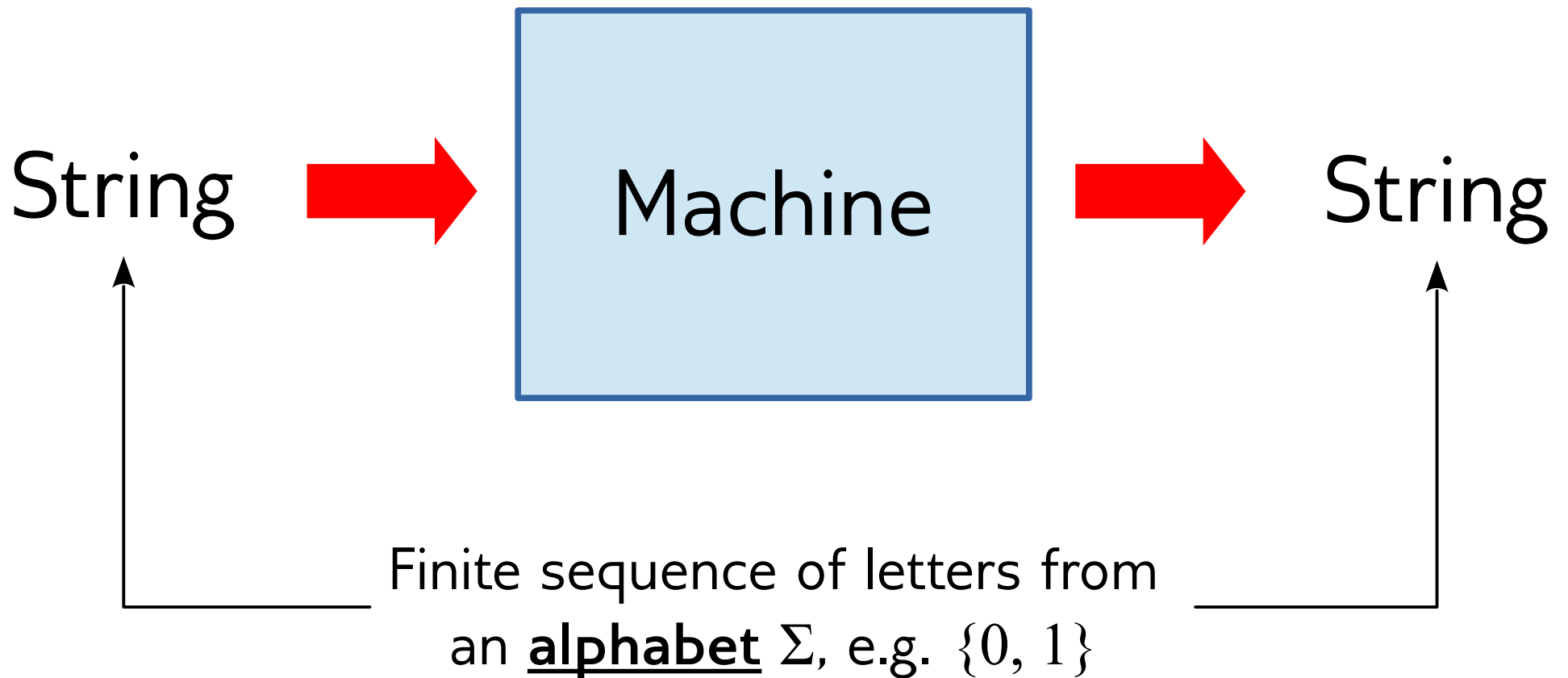
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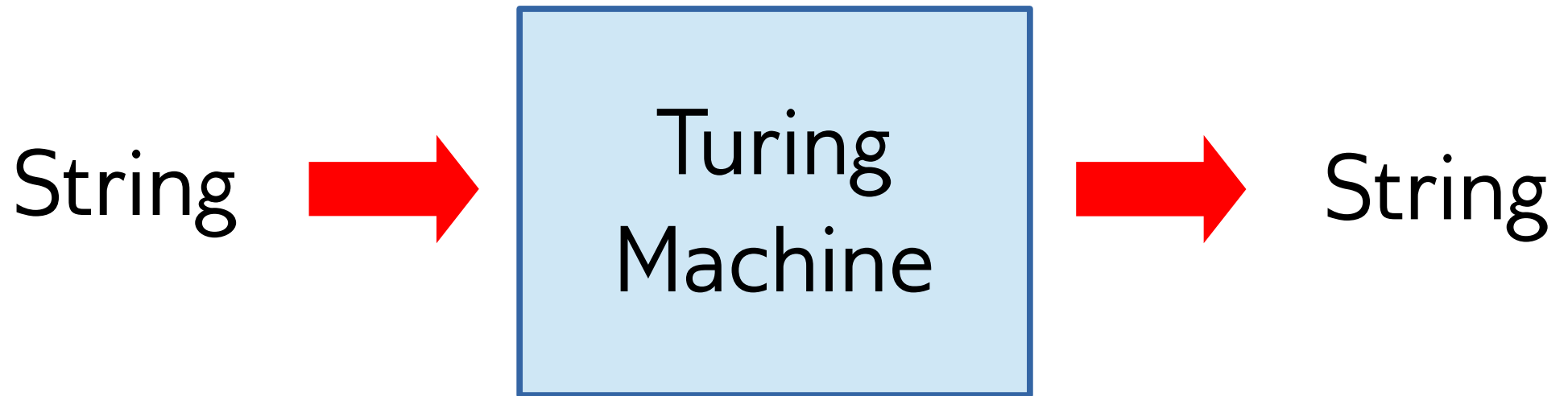
# A simplified model



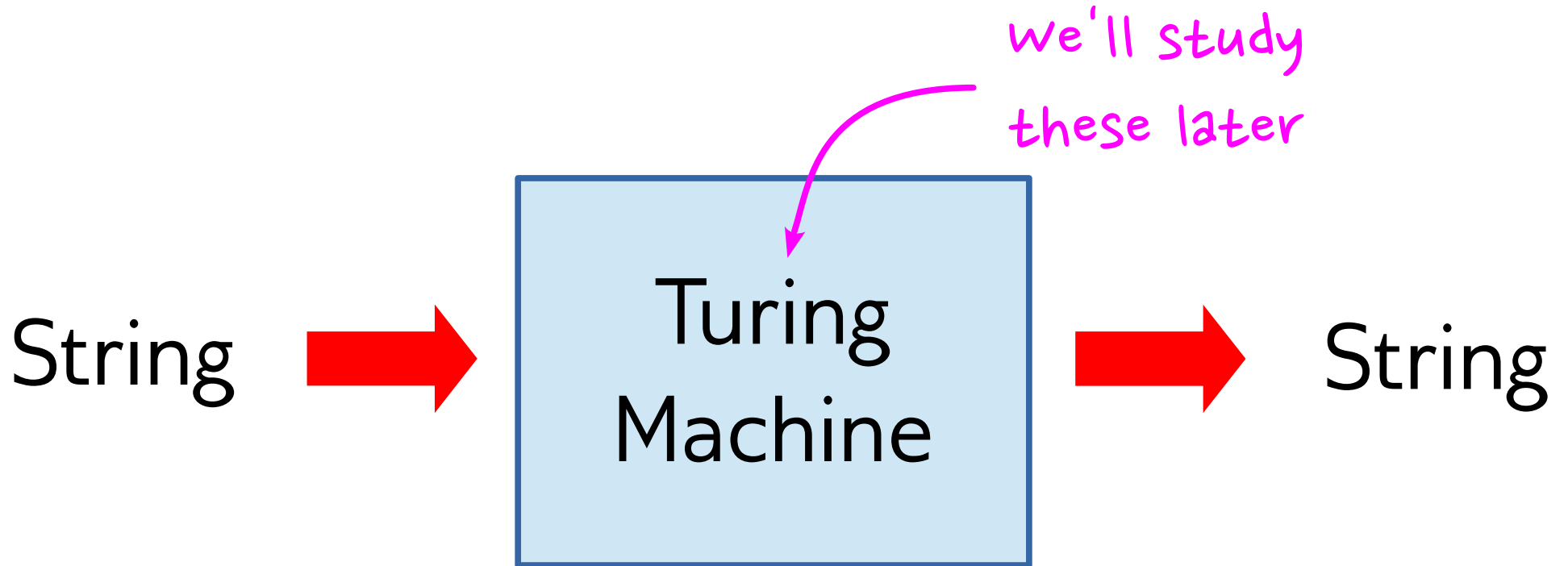
# A simplified model



# A general-purpose computer

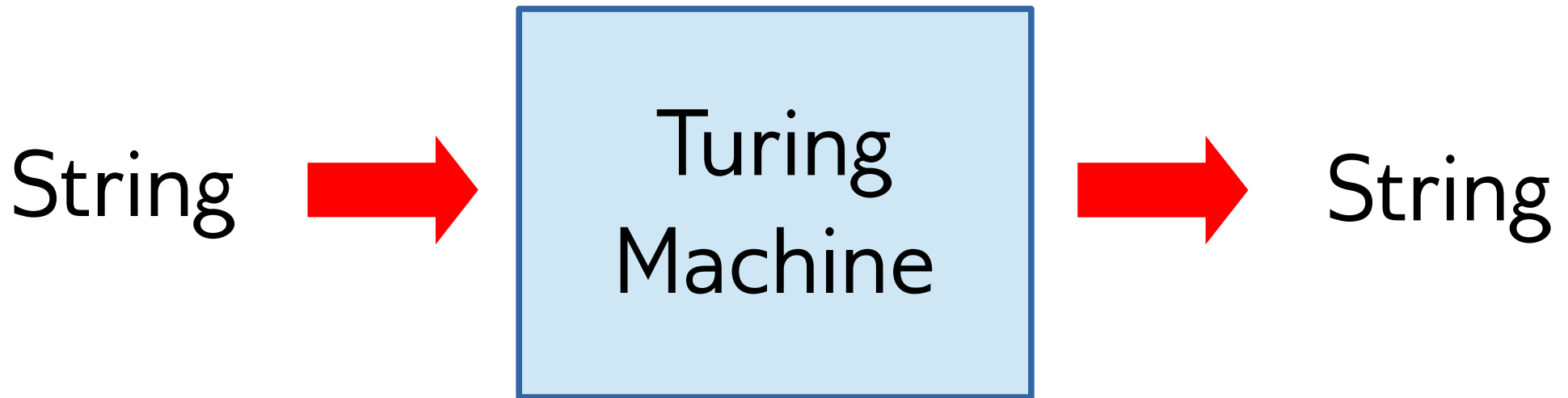


# A general-purpose computer





# A general-purpose computer

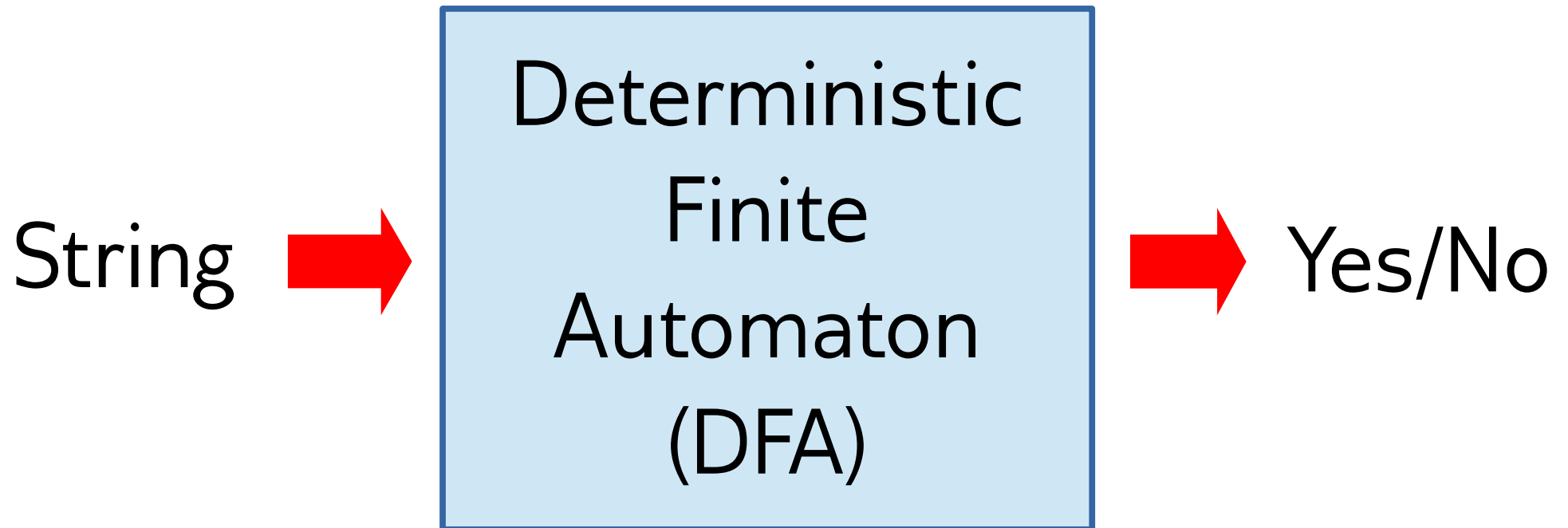


**Church-Turing Thesis:** Any “effective/mechanical/real-world” calculation can be carried out on a Turing machine

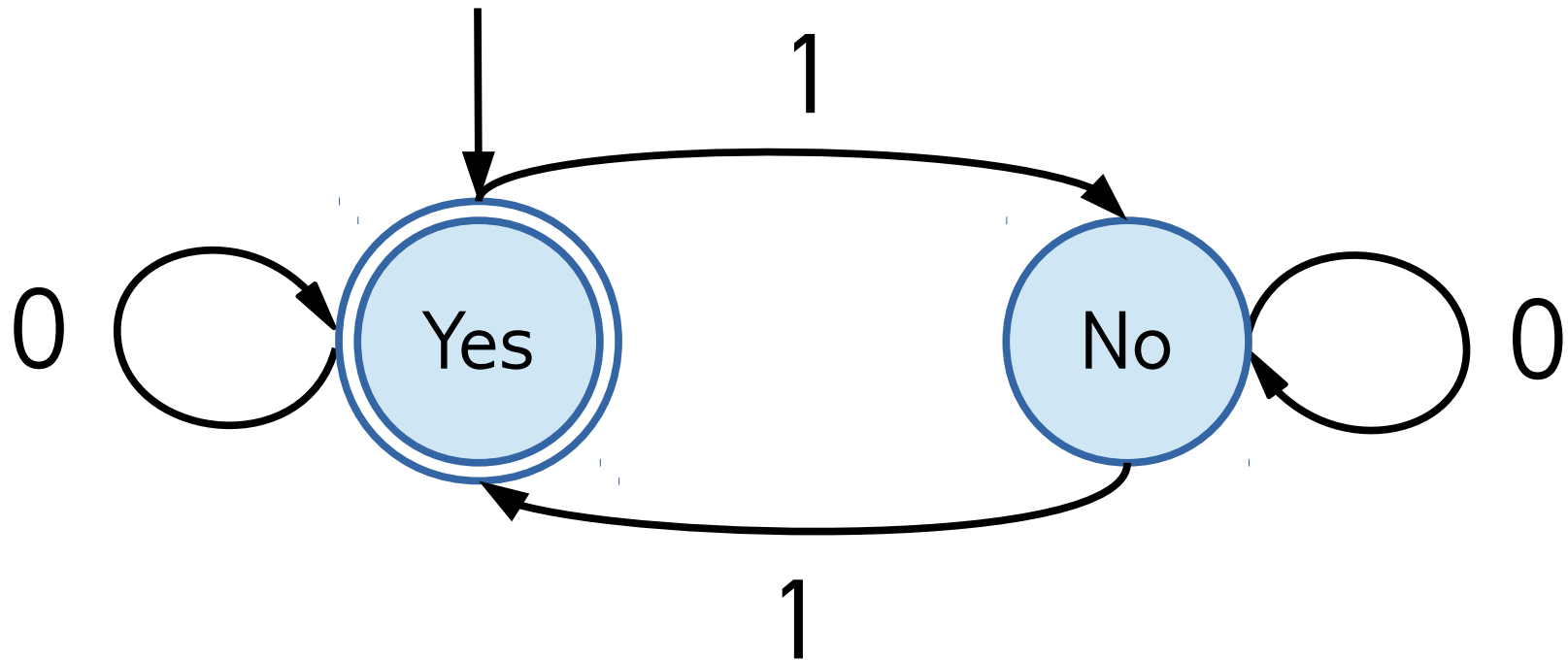


Alan Turing, 1912 - 1954

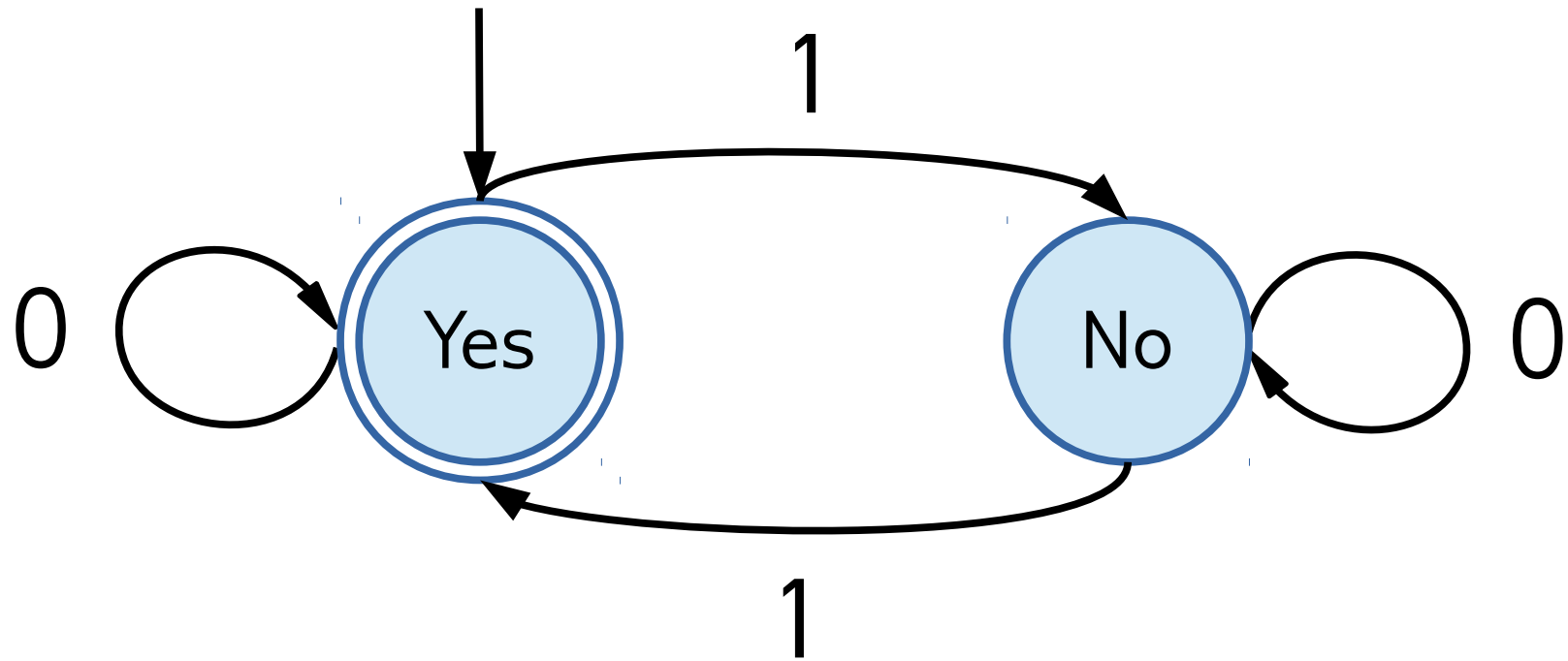
# A simple “computer”



# An example

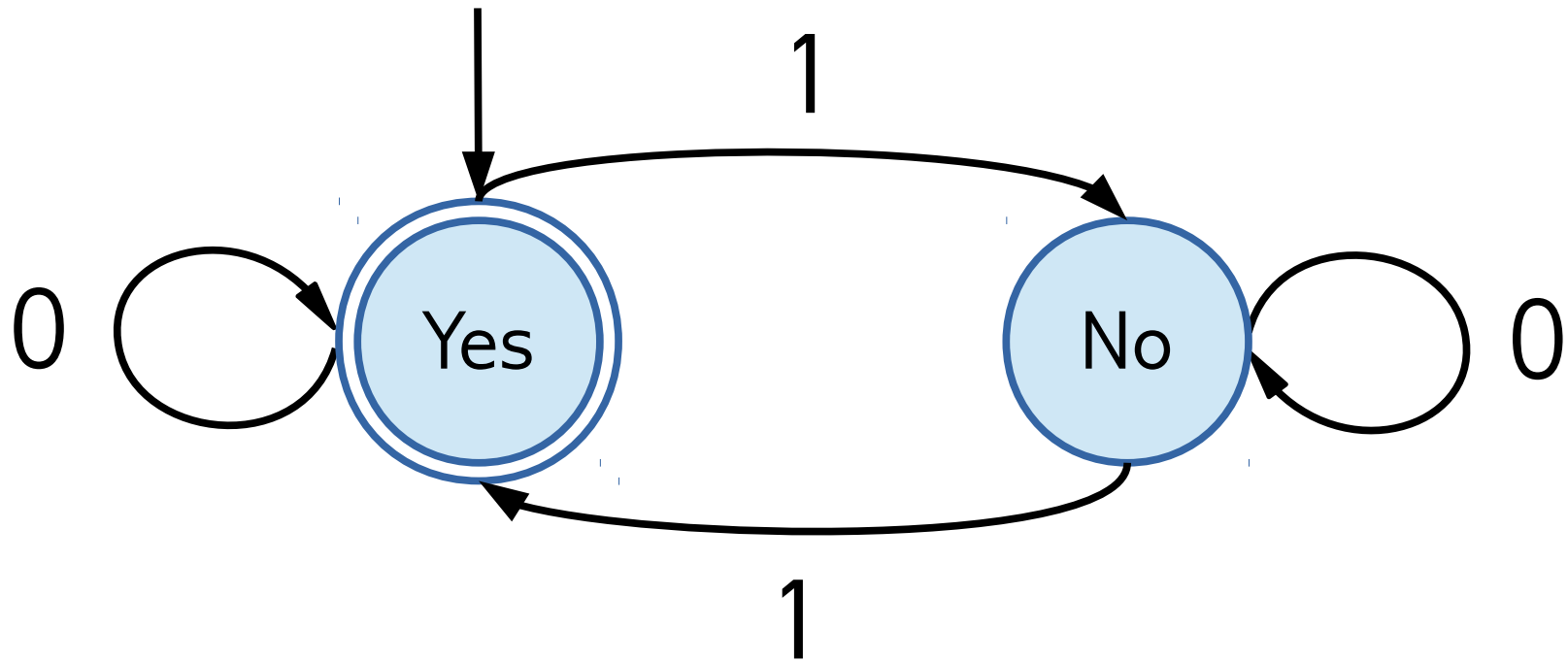


# An example



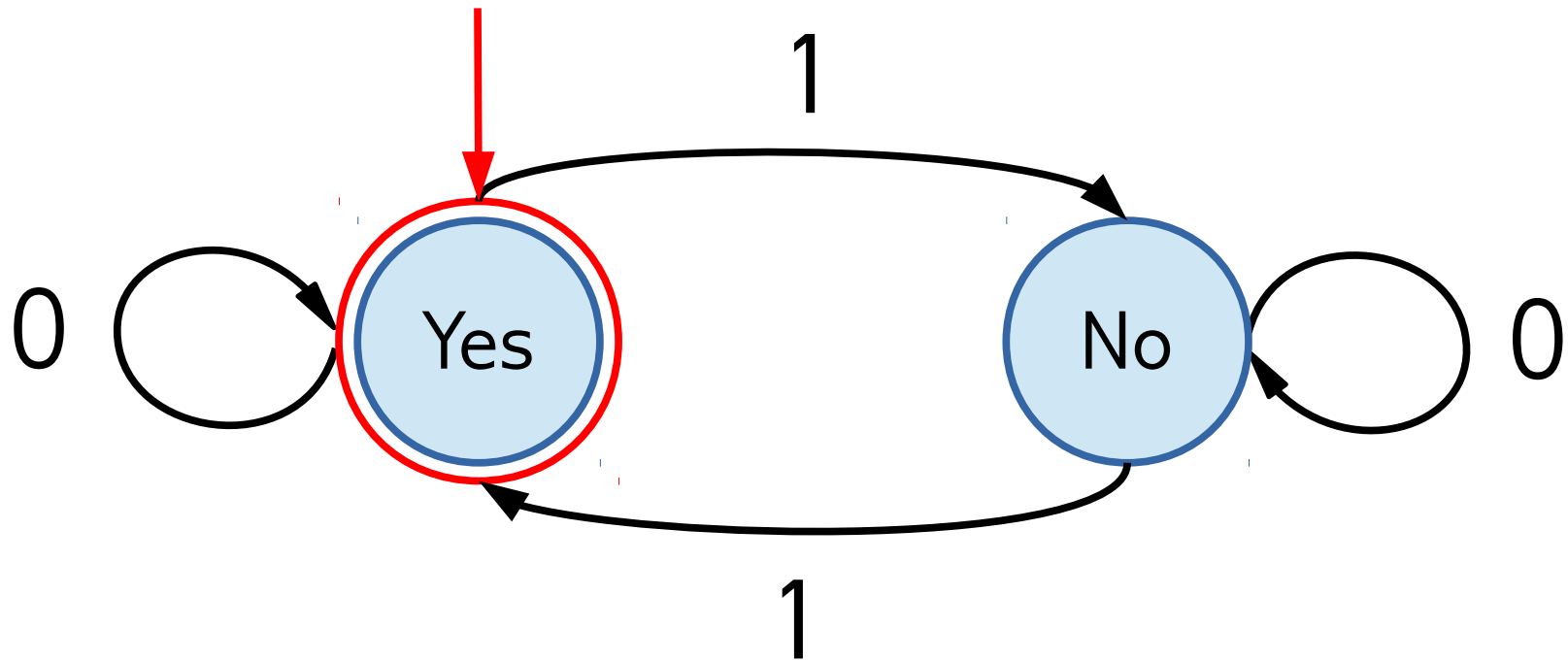
(Binary input)

# An example



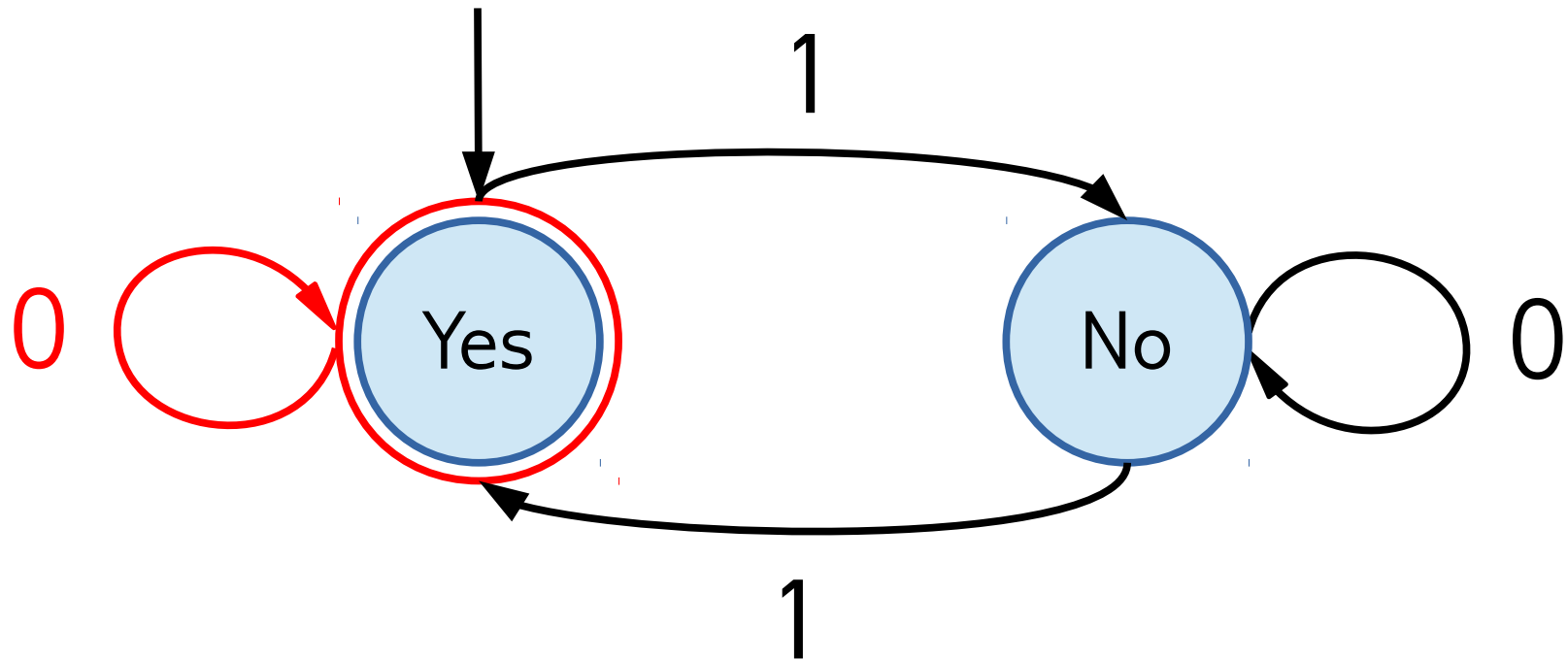
Input: 01001

# An example



Input: 01001

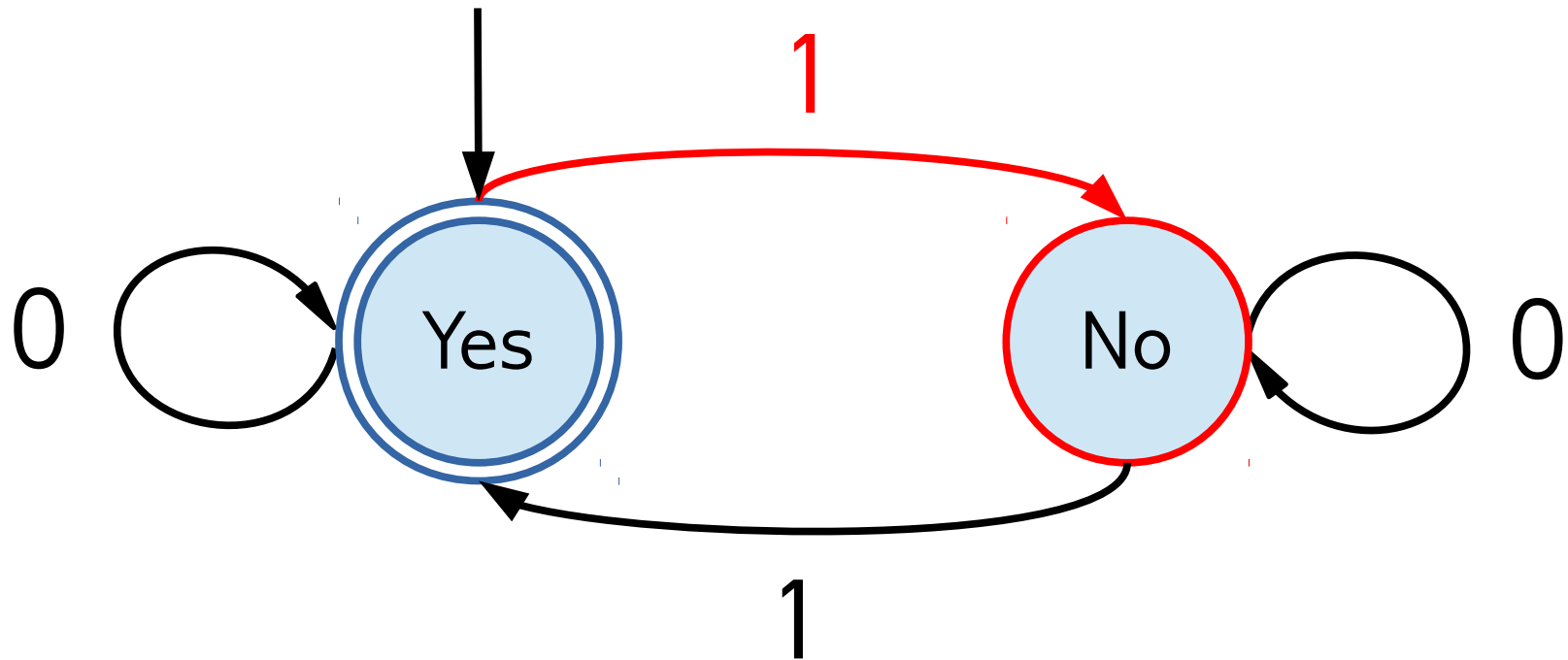
# An example



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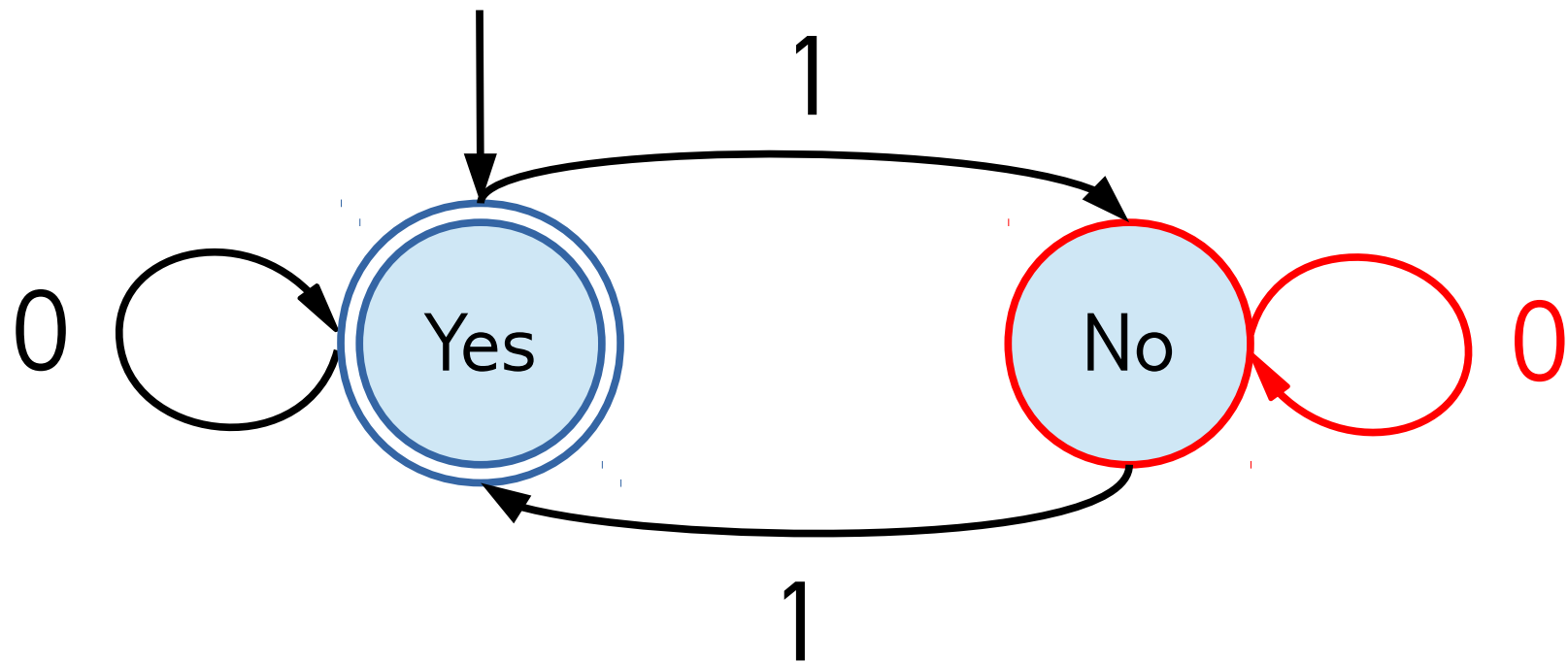


# An example



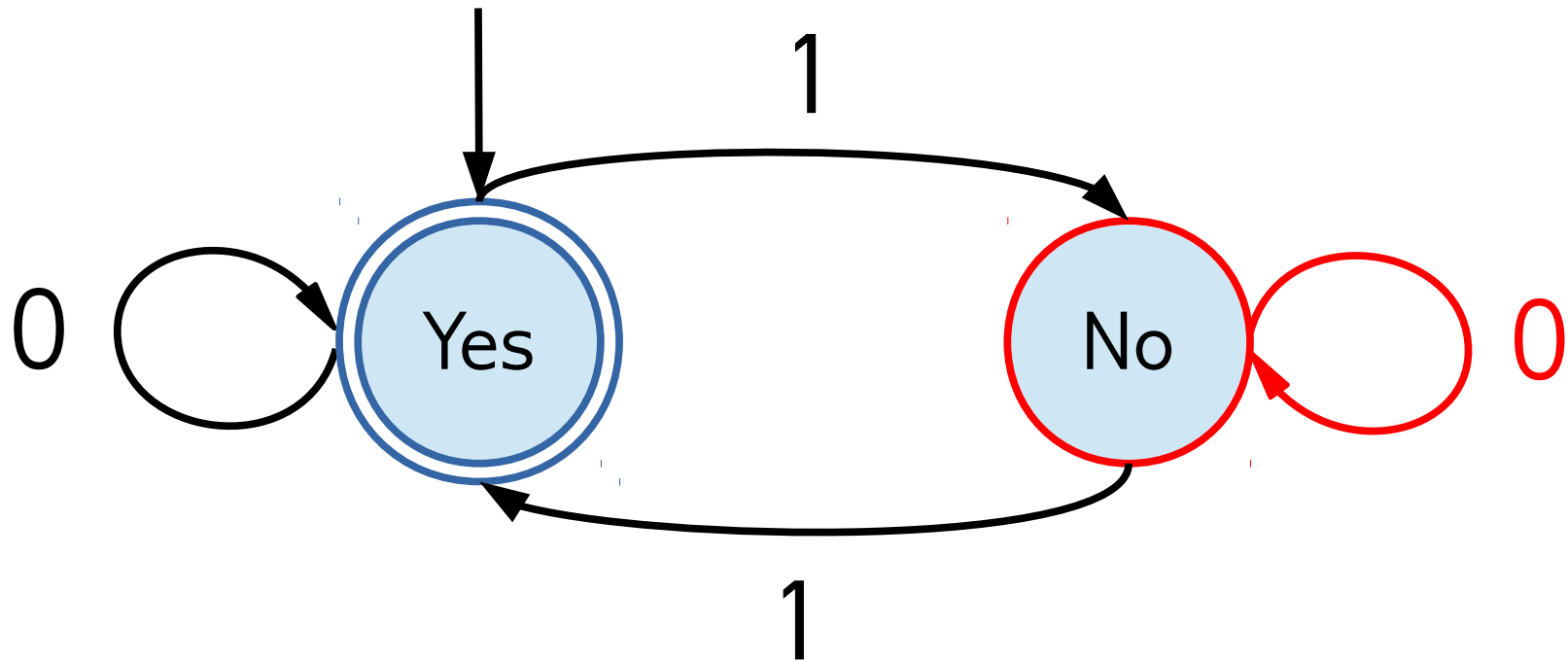
Input: 01001

# An example



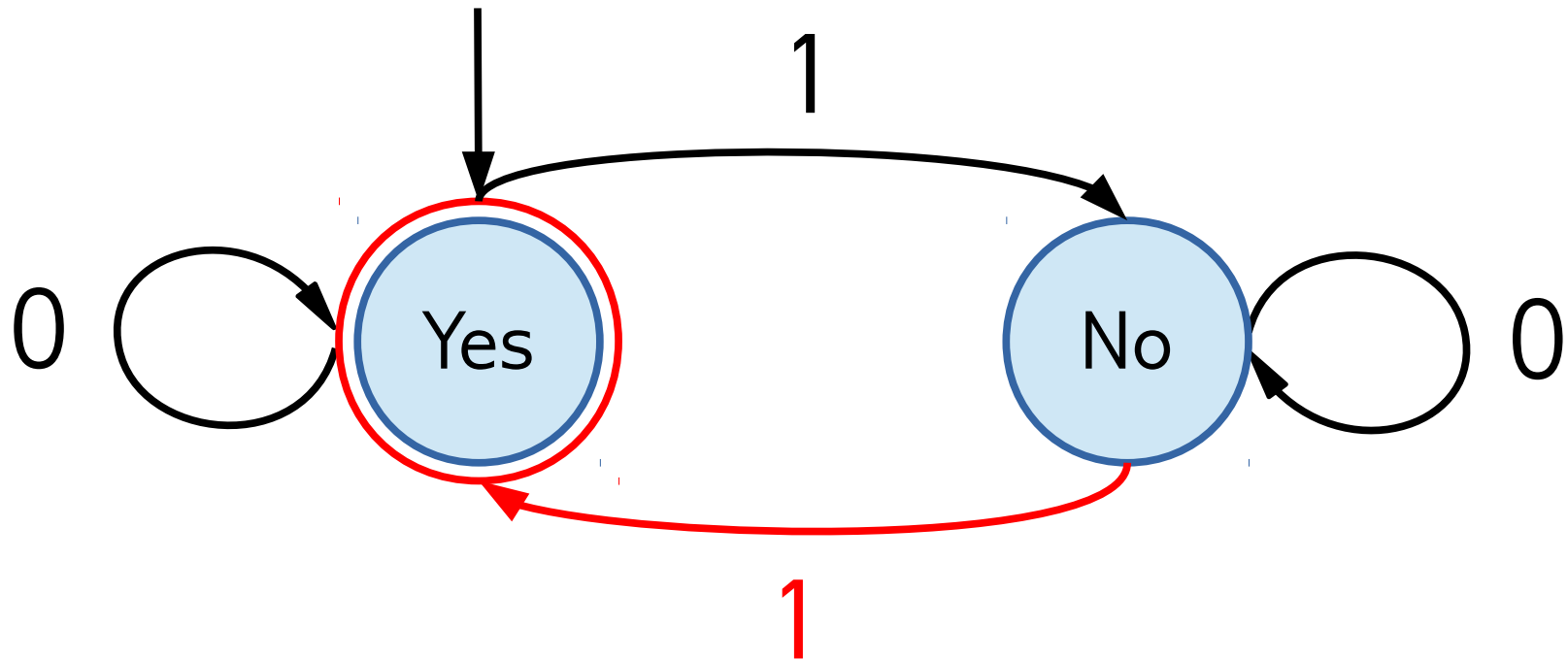
Input: 01001

# An example



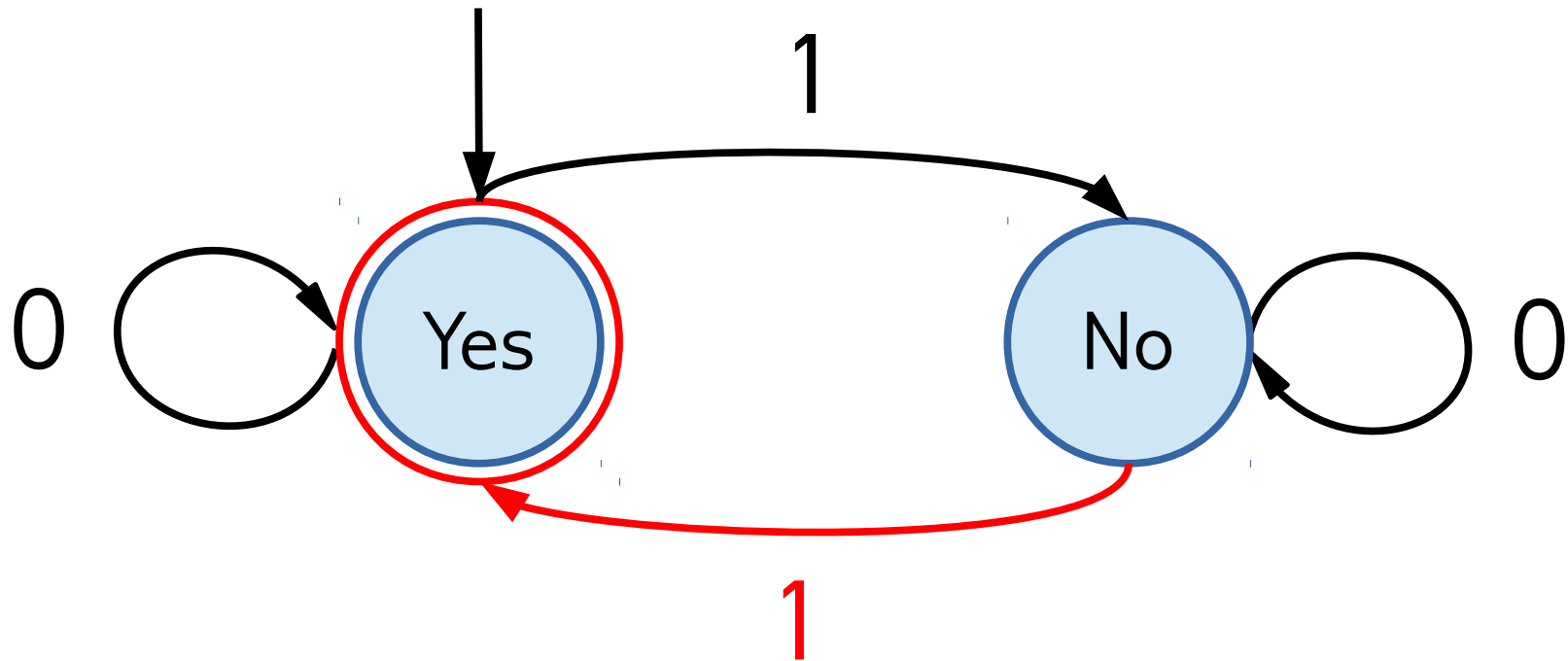
Input: 01001

# An example



Input: 0100**1**

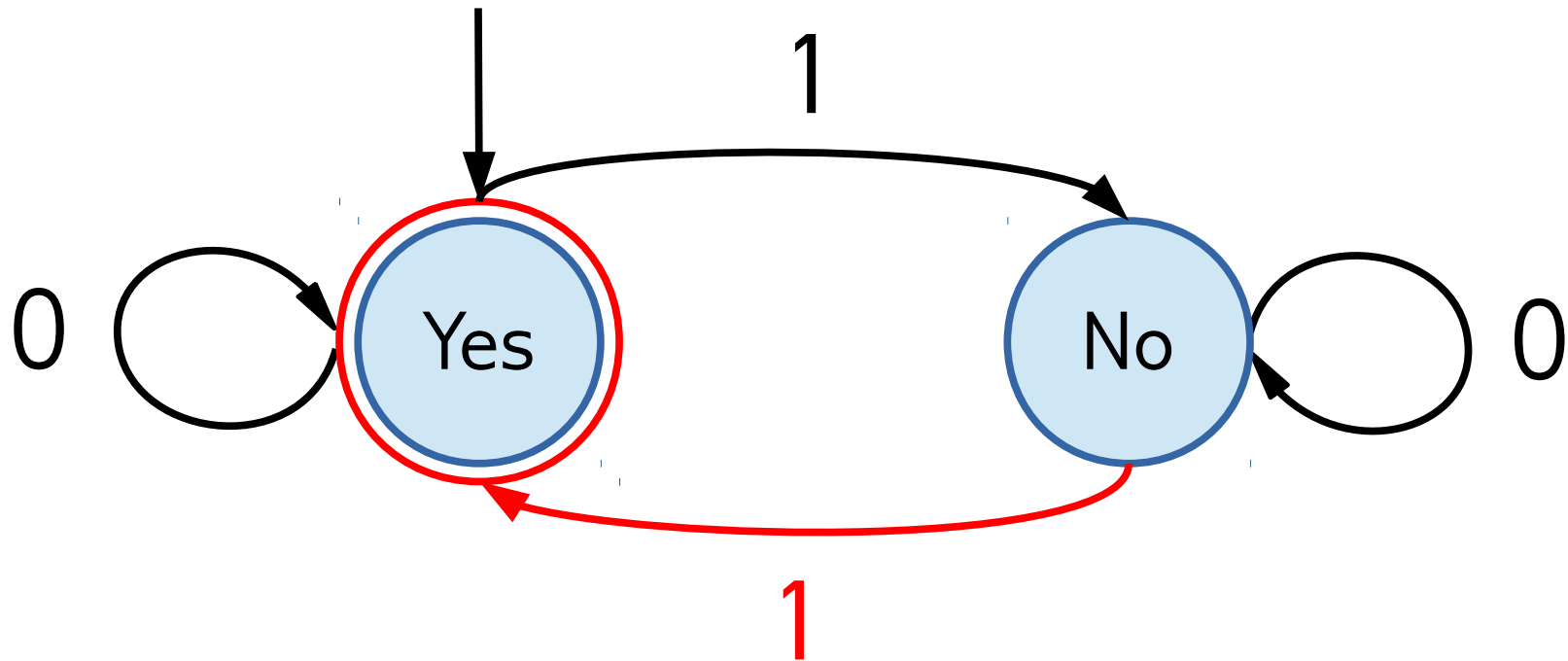
# An example



Input: 01001

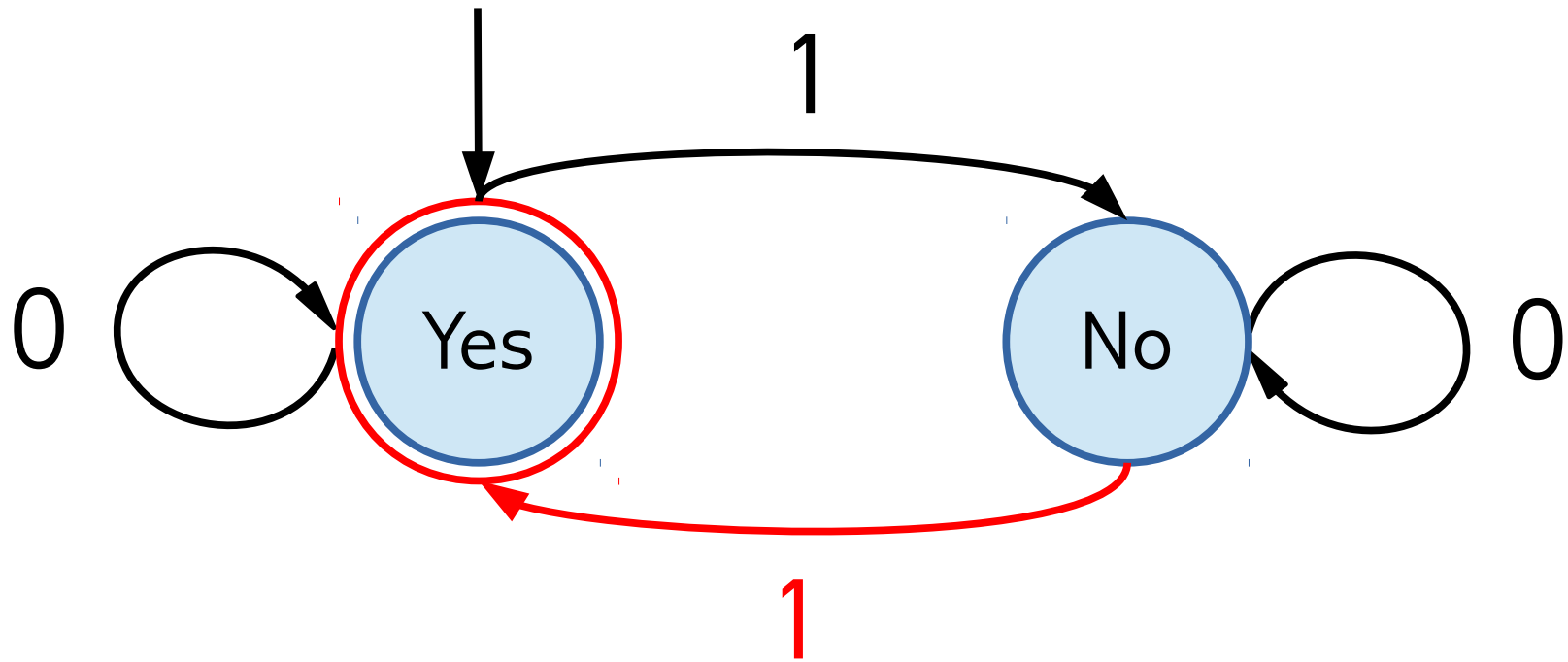
Output: **Yes!**

# An example



In general, on what binary strings does this DFA return Yes?

# An example



*Ans:* All strings with an even number of 1's

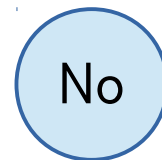
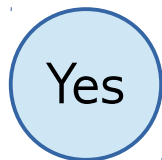
# Deterministic Finite Automaton

- A DFA is a 5-tuple  $M = (Q, \Sigma, \delta, q_0, F)$



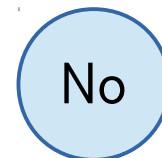
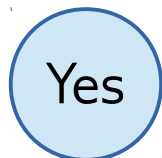
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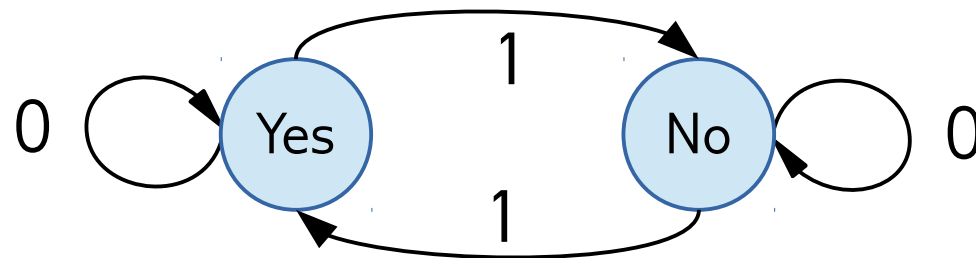
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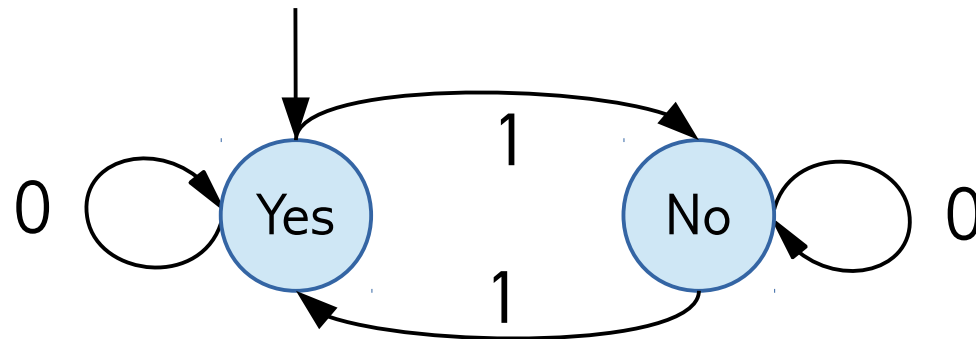
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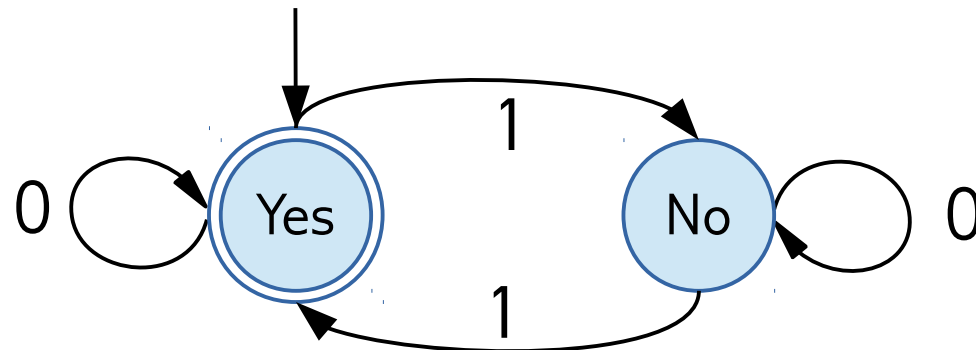
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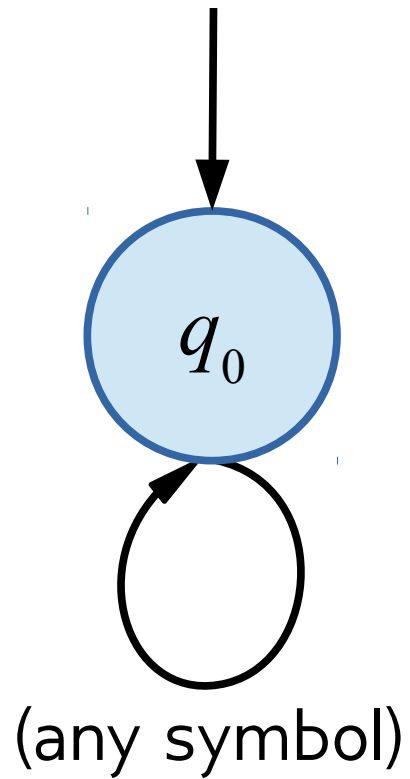


# Deterministic Finite Automaton

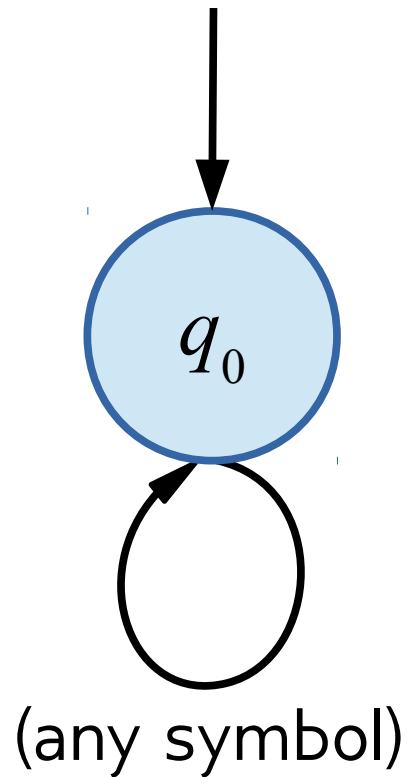
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  - $q_0 \in Q$  is the **start/initial state**
  - $F \subseteq Q$  is the set of **final/accepting states**



What does this DFA accept?

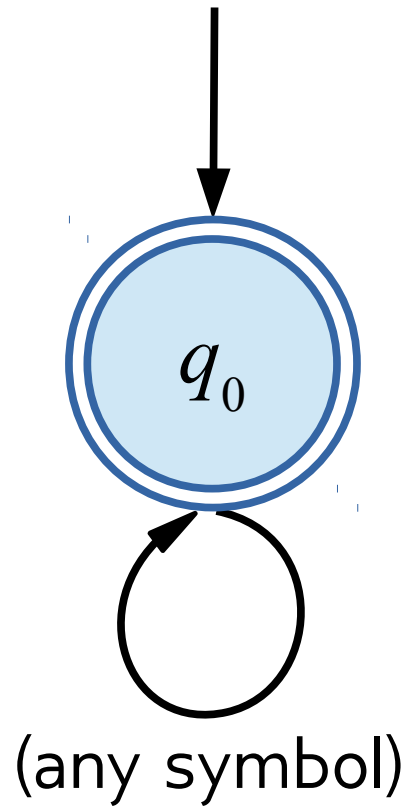


What does this DFA accept?



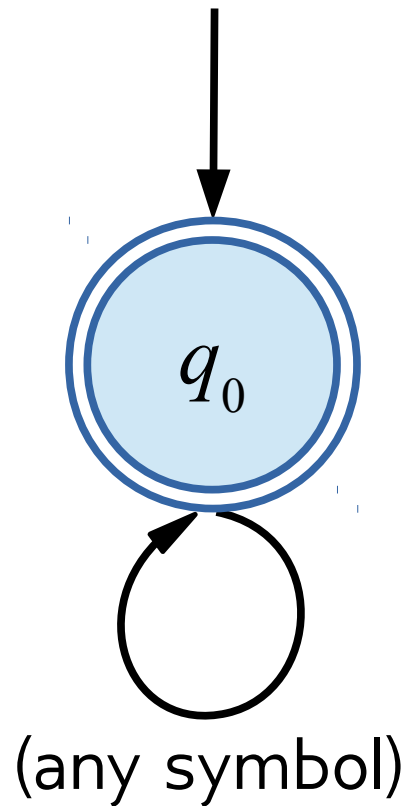
Answer: No strings

What does this DFA accept?



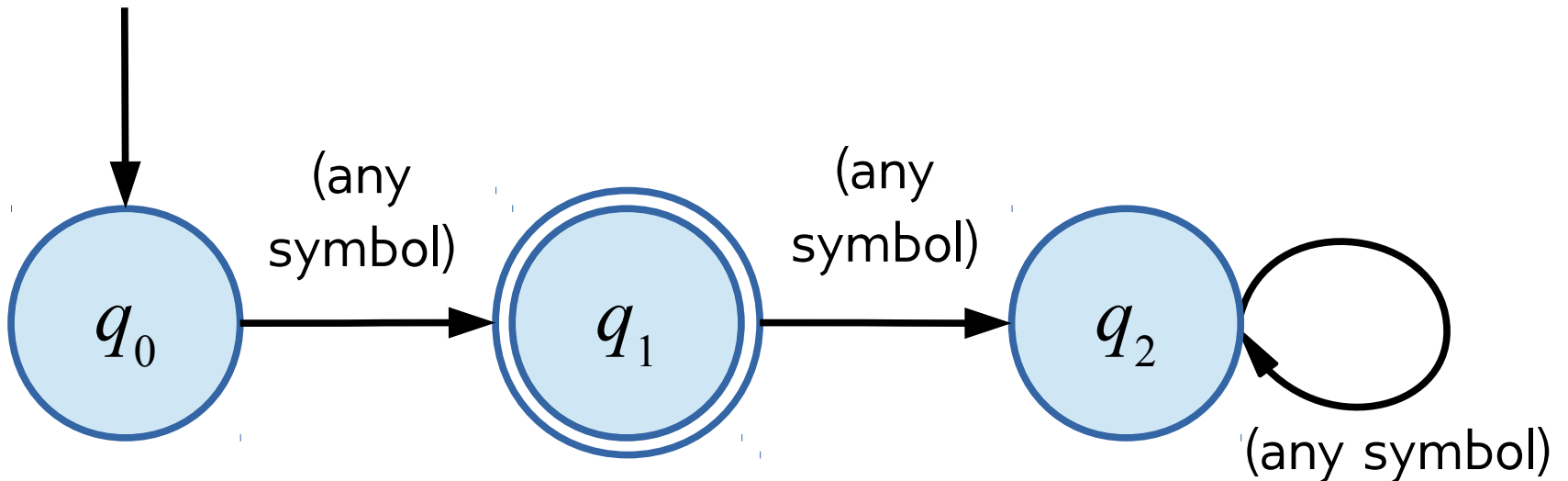


What does this DFA accept?

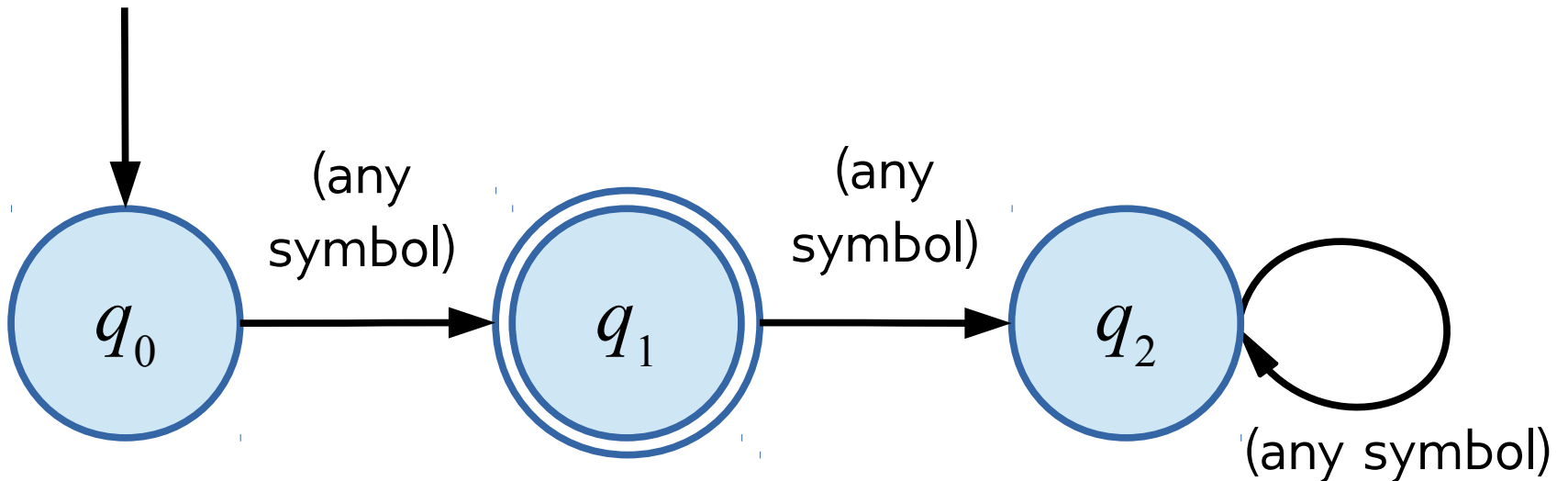


Answer: All strings

# What does this DFA accept?

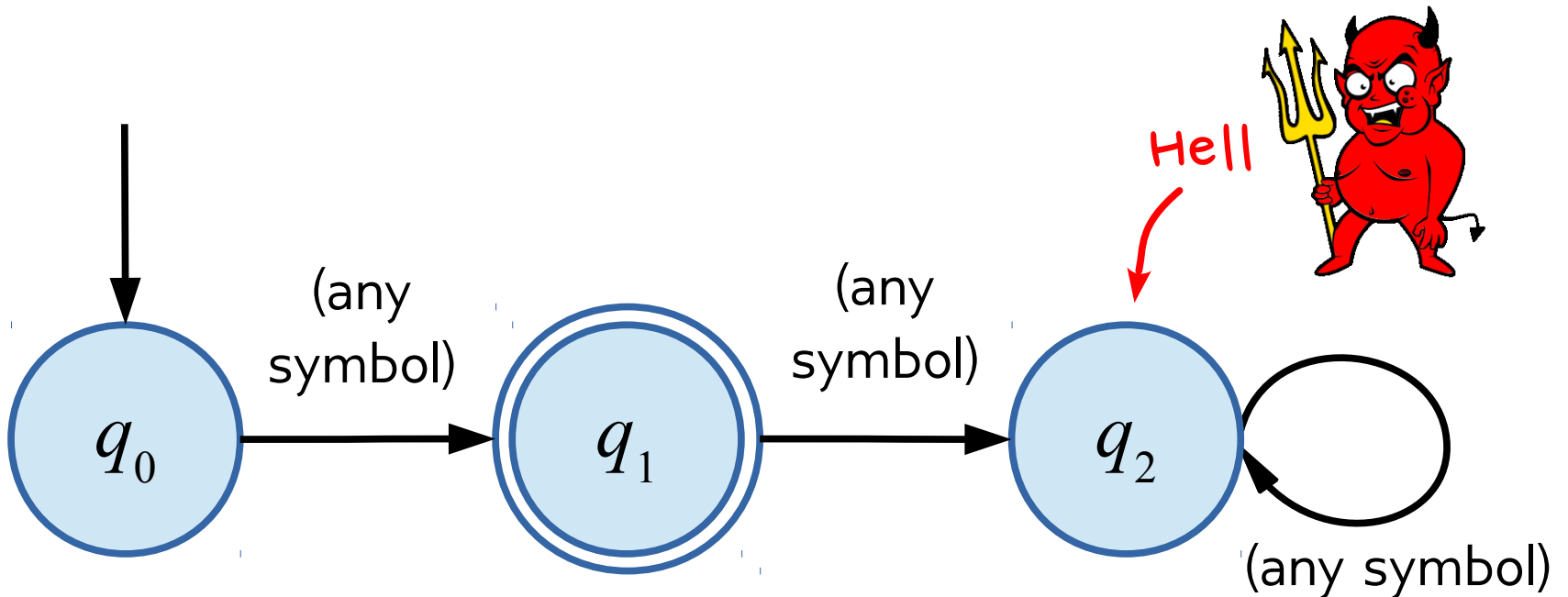


# What does this DFA accept?



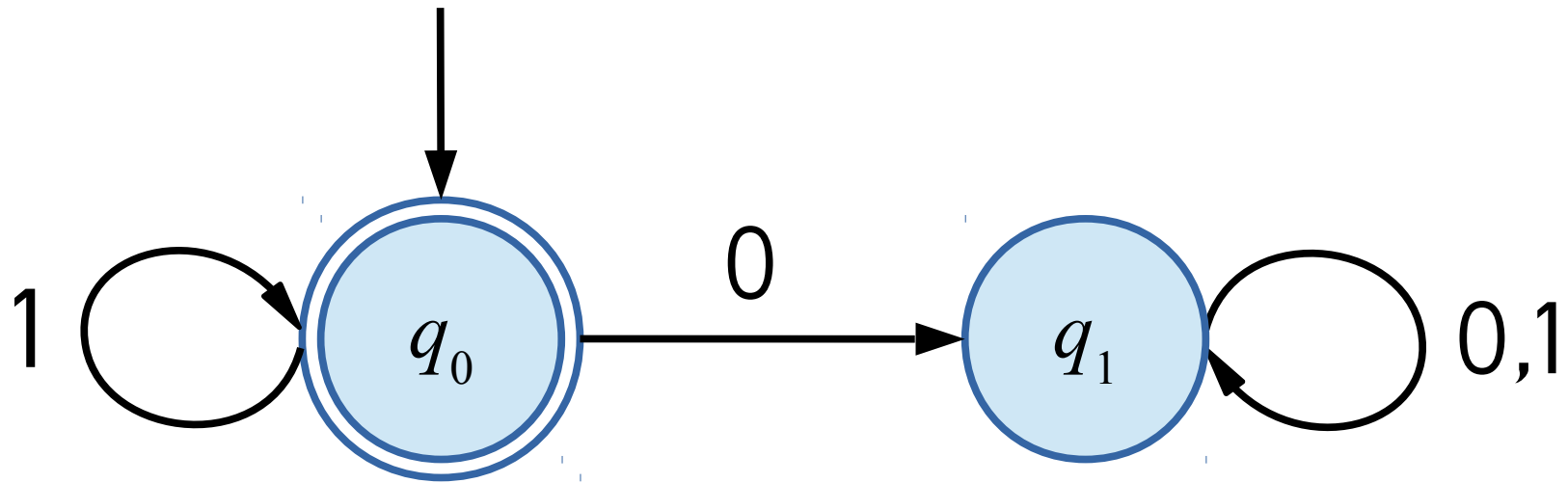
Answer: Strings of length 1

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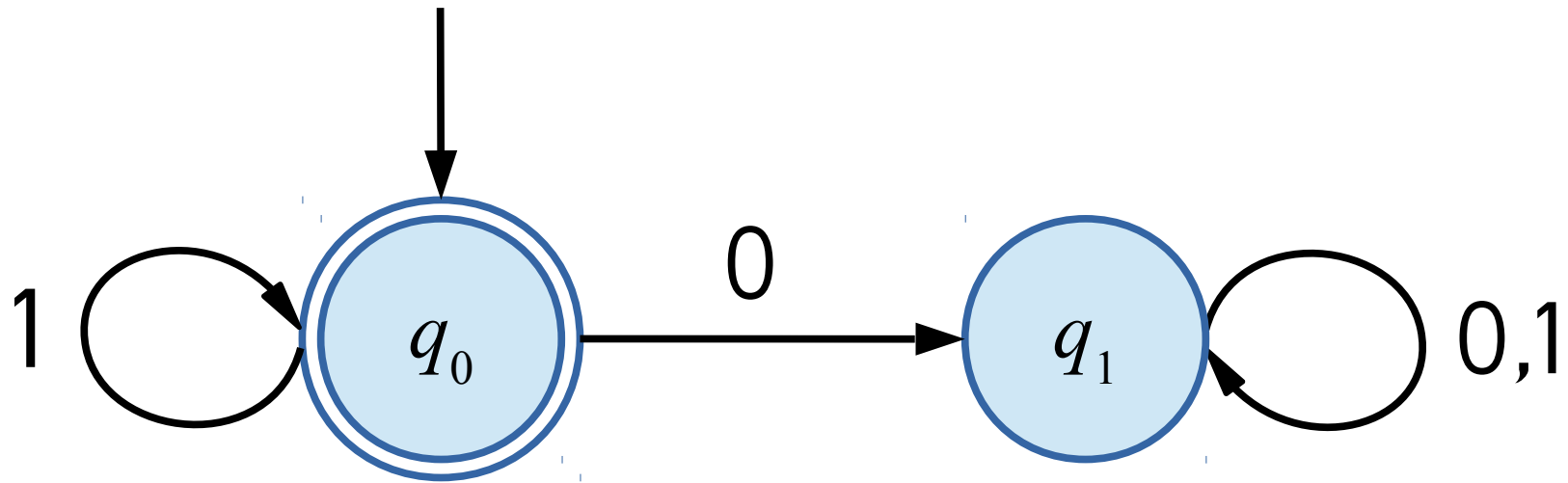


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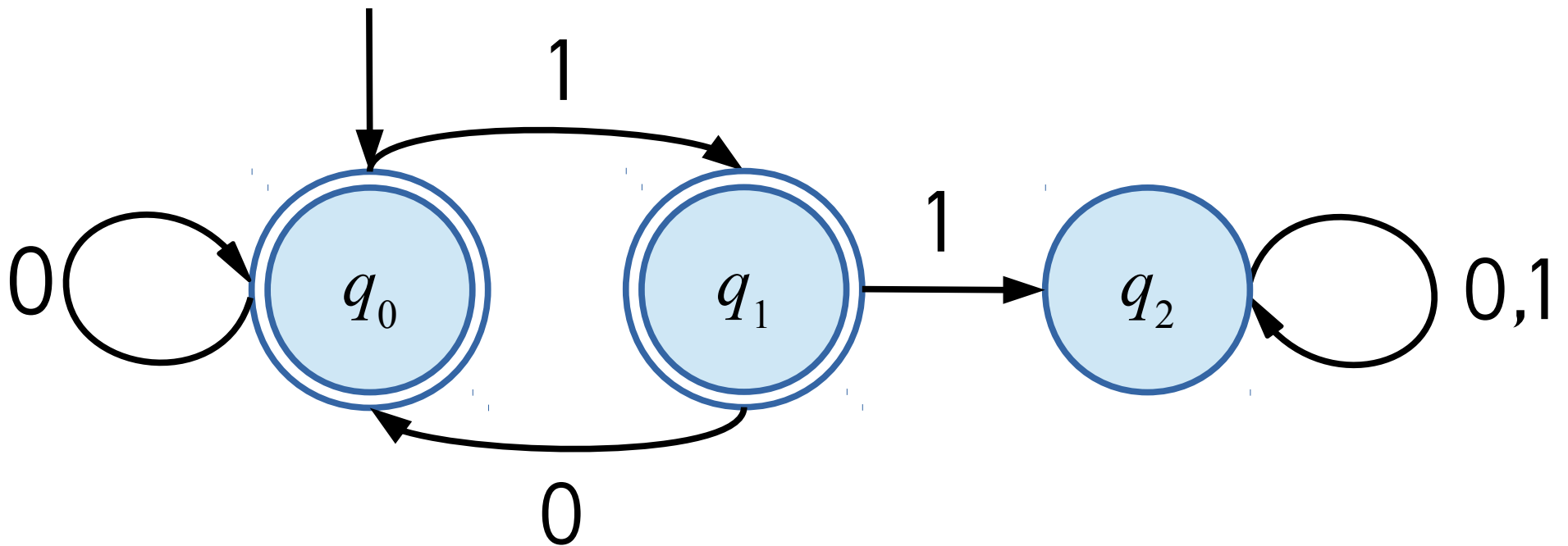


What does this DFA accept?

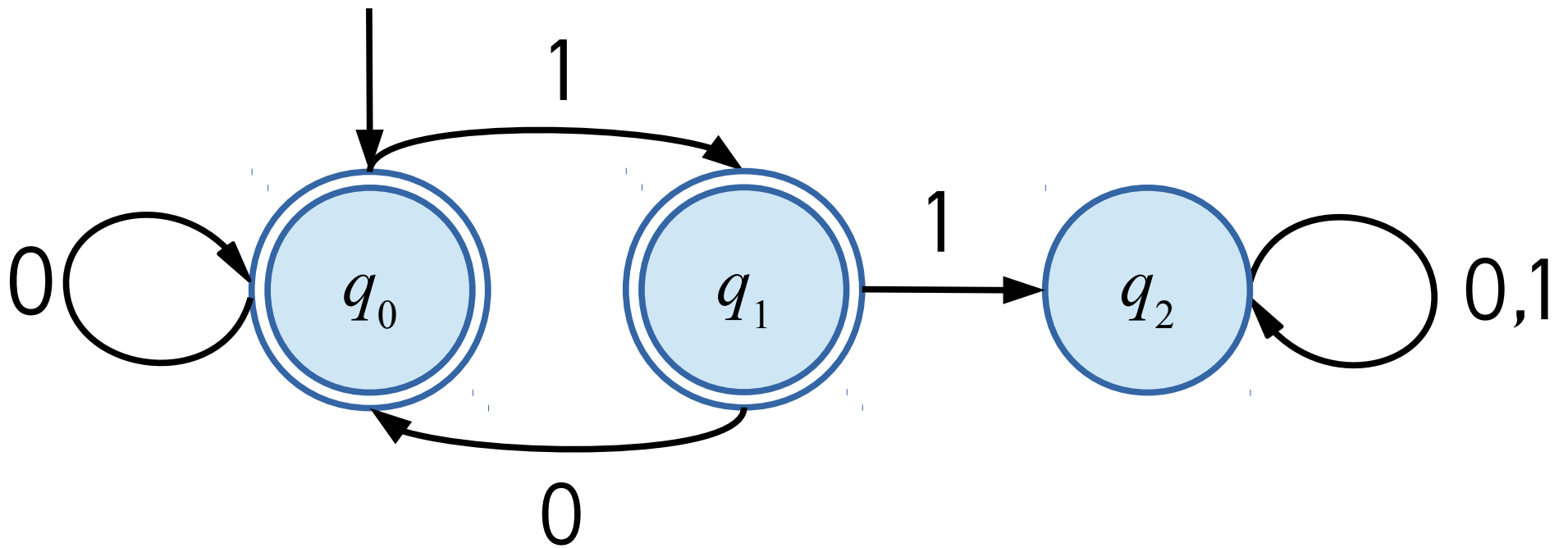


Answer: Strings containing only 1's

What does this DFA accept?



What does this DFA accept?




Answer: Strings containing  
no two consecutive 1's



# Language

- Given alphabet  $\Sigma$ , a **language**  $L$  is a set of strings over the alphabet, i.e.  $L \subseteq \Sigma^*$

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Set of all possible strings over  $\Sigma$
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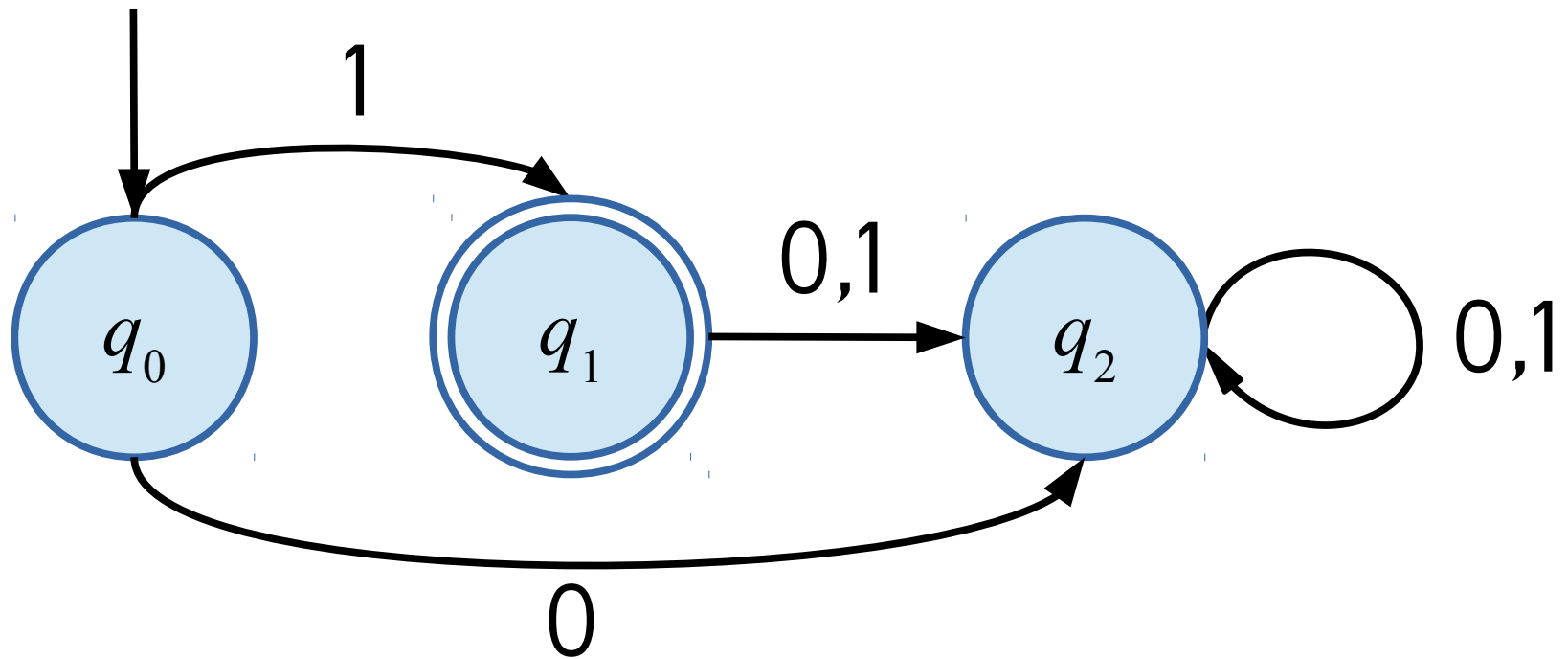


# Language

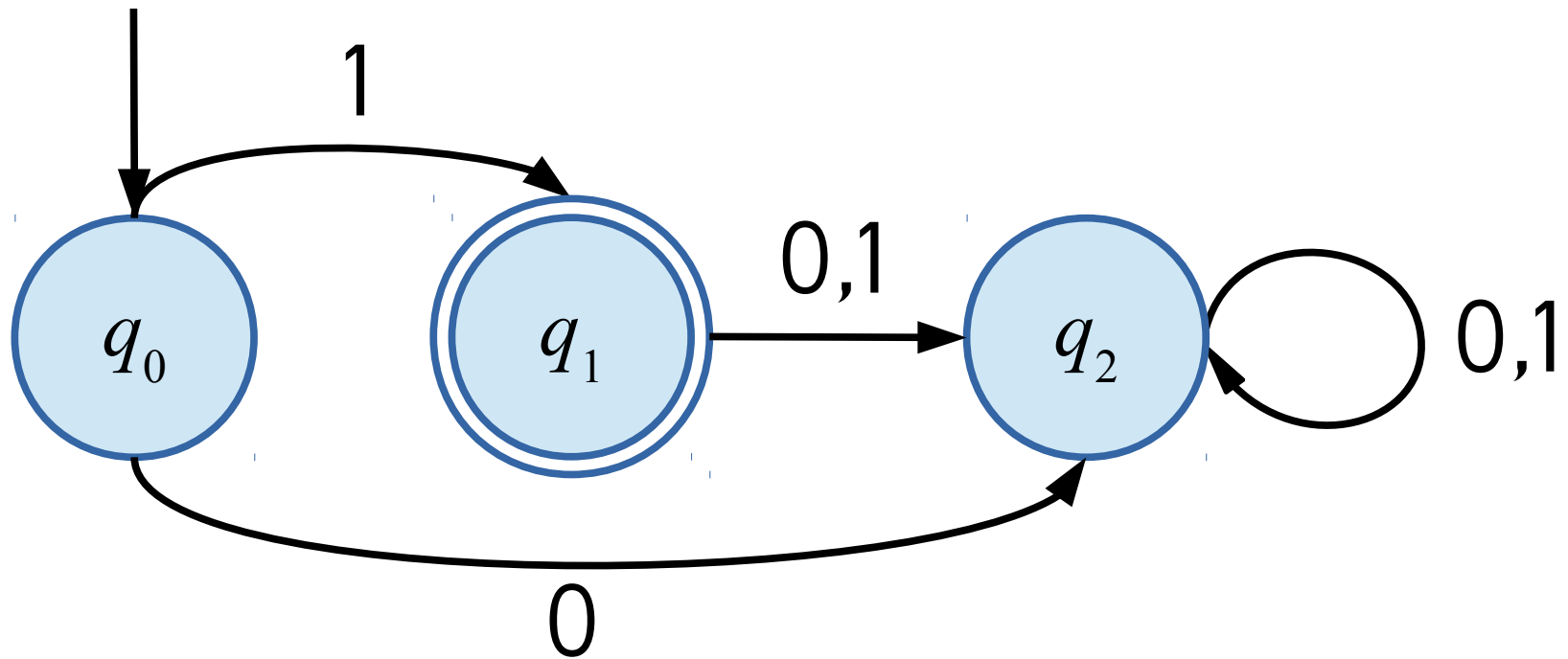
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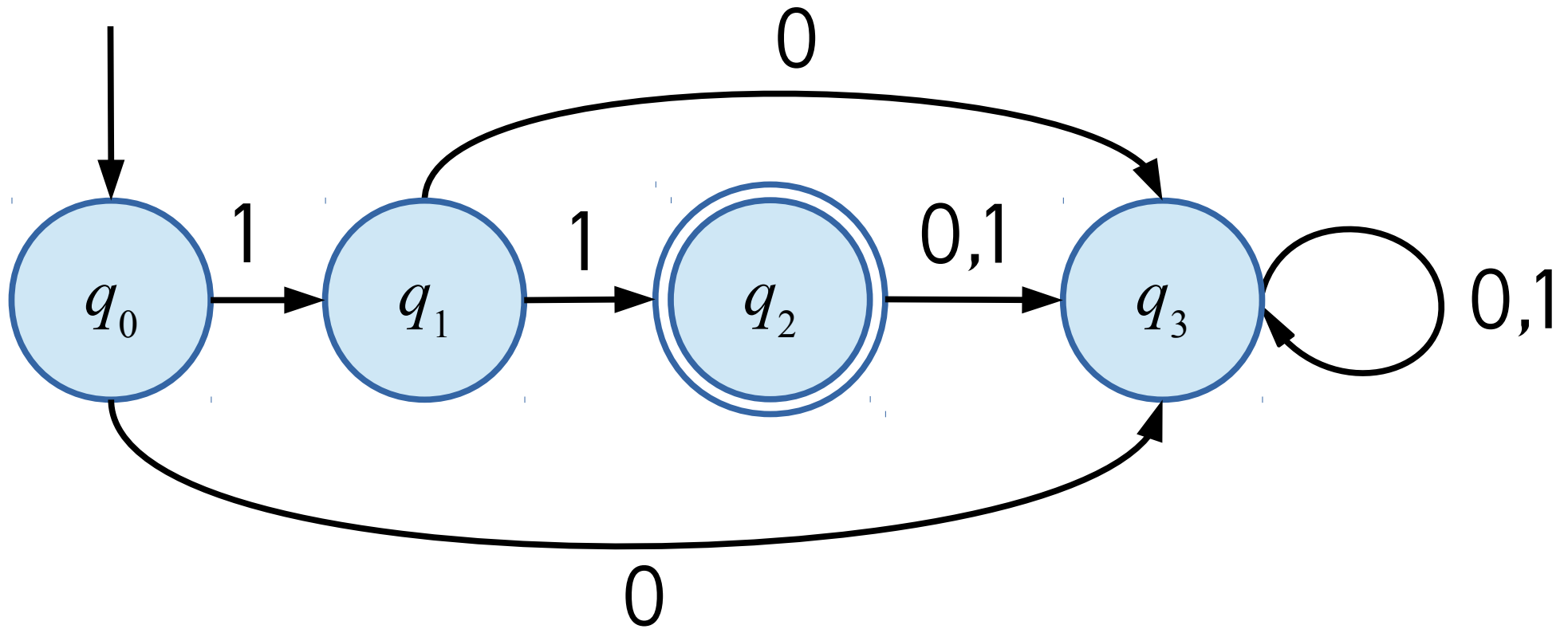


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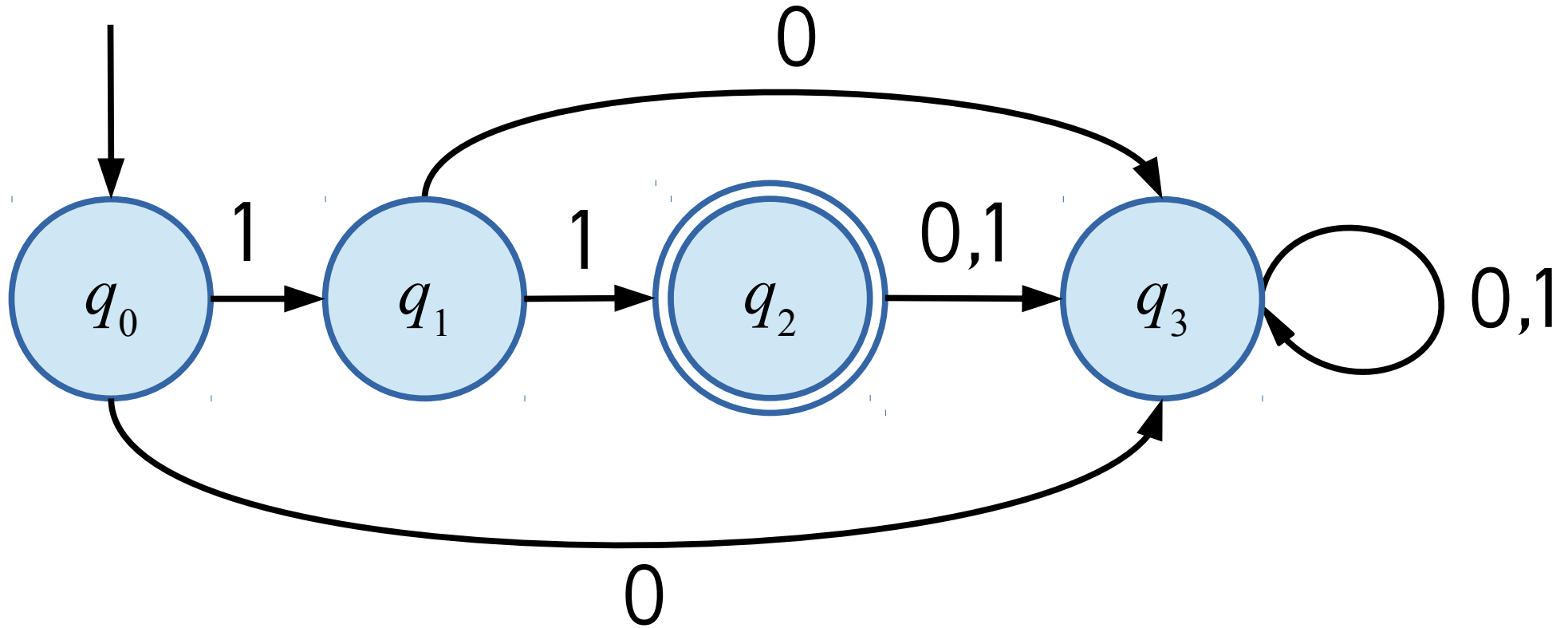


Answer: Only the string 1

What language does this DFA accept?



What language does this DFA accept?



Answer: Only the string 11



# DFA's find it difficult to count

- A DFA that recognizes the language  $\{1^c\}$  (the single string of  $c$  1's) must have at least  $c$  states

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- A DFA that recognizes the language  $\{1^c\}$  (the single string of  $c$  1's) must have at least  $c$  states
  - The parent alphabet is irrelevant (but must of course contain 1)

(Proof discussion to be completed next class)