Proofs

CS 2800: Discrete Structures, Fall 2014

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Thought for the Previous Day

Is this statement true or false?

"All eleven-legged alligators have orange and blue spots"

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Thought for the Previous Day

This statement is ***true***!!!

... else there would be an eleven-legged alligator
(which lacks orange spots, or blue spots, or both)

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Recap: left and right inverses

- $g : B \longrightarrow A$ is a left inverse of $f : A \longrightarrow B$ if $g(f(a)) = a$ for all $a \in A$
- $h : B \longrightarrow A$ is a right inverse of $f : A \longrightarrow B$ if $f(h(b)) = b$ for all $b \in B$

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Recap: left and right inverses

- A function is ***injective*** (one-to-one) iff it has a ***left inverse***
- A function is ***surjective*** (onto) iff it has a ***right inverse***

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Thought for the Day #1

If a function has both a left inverse and a right inverse, then the two inverses are identical, and this common inverse is unique.

http://www.cs.cornell.edu/courses/cs2800/2014fa/handouts/jonpak_function_notes.pdf

Bijection and two-sided inverse

- Two-sided inverse of $f: A \longrightarrow B$ is a function $g: B \longrightarrow A$ that is both a left inverse and a right inverse
- A function f is bijective iff it has a two-sided inverse

Proof worked out on board here, also see
http://www.cs.cornell.edu/courses/cs2800/2014fa/handouts/jonpak_function_notes.pdf

A proof from set theory

For any two sets A and B,

$$A = (A \cap B) \cup (A \setminus B)$$

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Definition: Set equality

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

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Definitions: Set operators

- $A \cup B$ consists of all elements in A or B (or both!)
- $A \cap B$ consists of all elements in both A and B
- $A \setminus B$ consists of all elements in A but not in B
- A' consists of all elements not in A

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A proof from set theory

For any two sets A and B,

$$A = (A \cap B) \cup (A \setminus B)$$

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Another proof from set theory

For any two sets A and B,

$$A \setminus B = A \cap B'$$

Prove this on your own!

Benjamin D. Esham

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XKCD

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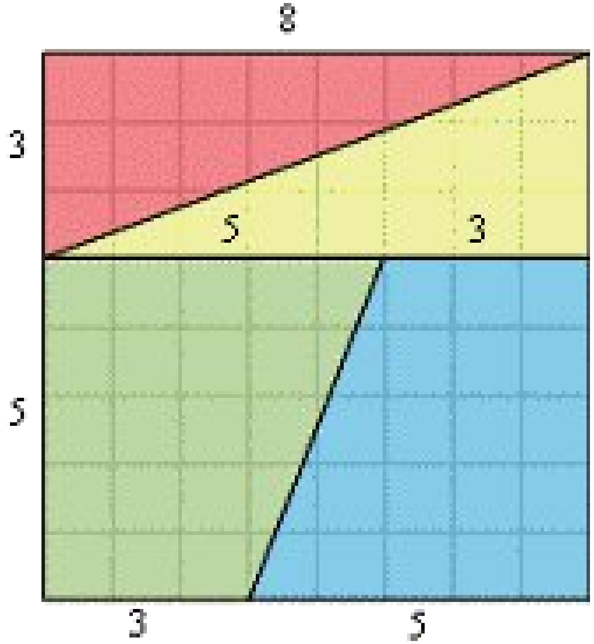

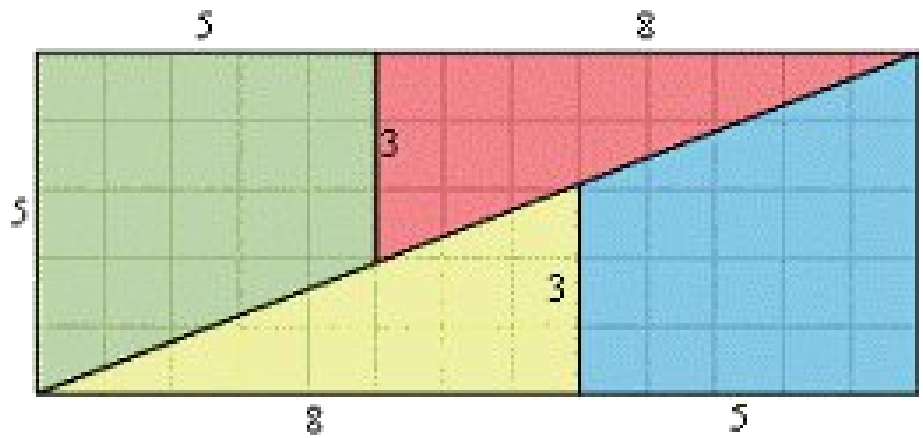

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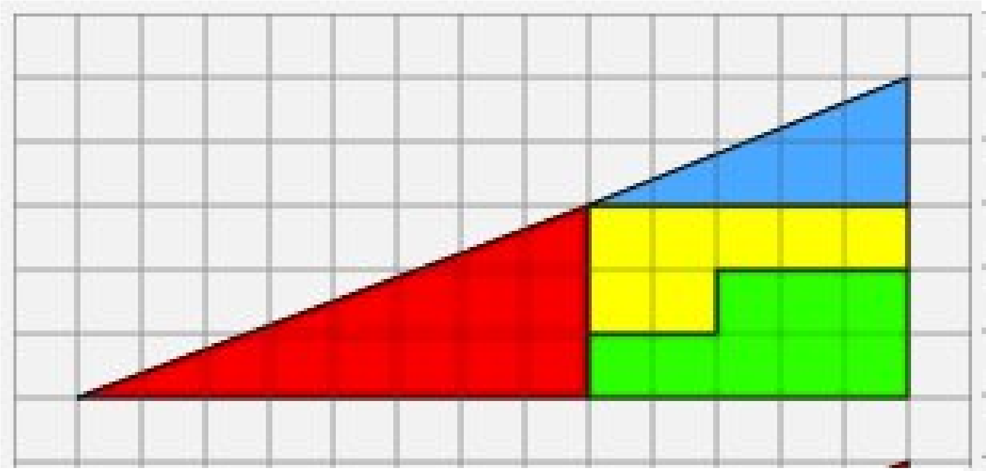

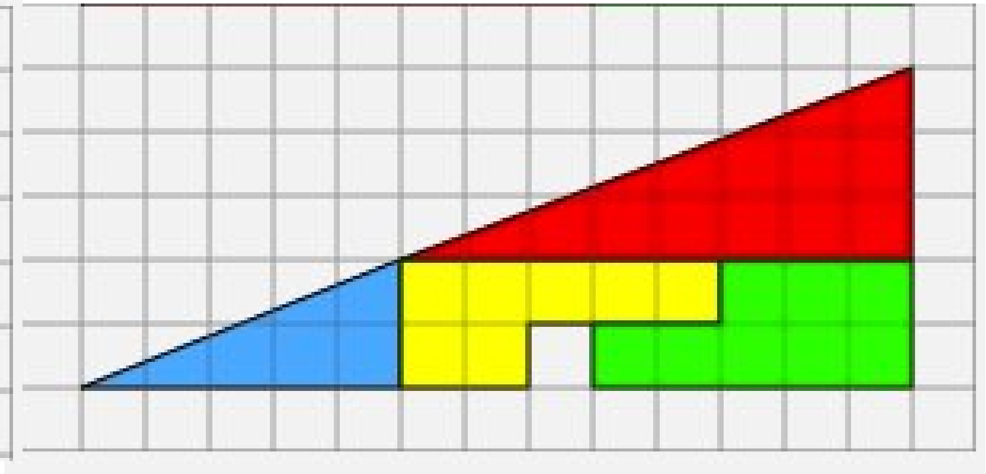

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Let $a = b = 1$ ...

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