Functions and Quantifiers

CS 2800: Discrete Structures, Fall 2014

Sid Chaudhuri


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COMMENTATOR ALTON BROWN
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A chef is a function*

$$chef: 2^{Ingredients\ **} \rightarrow Dishes$$

* Assuming a chef always makes a single dish with each set of ingredients (else (s)he is a relation)

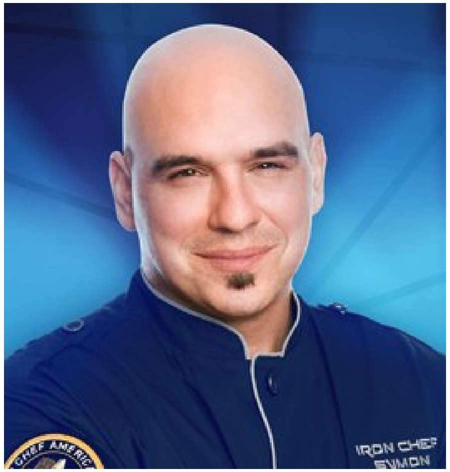
** $2^{Ingredients}$ is the power set of *Ingredients*

Bobby

Cat

Masaharu

Jose



Michael

Mario

A set of functions

$$\mathit{Chefs} = \{\ \mathit{bobby}, \mathit{cat}, \mathit{masaharu}, \mathit{jose}, \mathit{michael}, \mathit{mario}, \ldots\ \}$$

- Each chef maps subsets of ingredients to dishes
  - $\mathit{bobby}: 2^{\mathit{Ingredients}} \to \mathit{Dishes}$
  - $\mathit{cat}: 2^{\mathit{Ingredients}} \to \mathit{Dishes}$
  - …
- Two chefs do not, in general, map the same ingredients to the same dish

Chefs for courses

$$favorite\colon Courses \to Chefs$$

$$favorite(appetizers) = bobby$$
$$favorite(dessert) = cat$$

$\ldots$

Or in excruciating detail...

$$\mathit{favorite}\colon X \to [Y \to Z]$$

$$X = \mathit{Courses}$$

$$Y = 2^{\mathit{Ingredients}}$$

$$Z = \mathit{Dishes}$$

Or in excruciating detail...

$$favorite \in [X \to [Y \to Z]]$$

$$X = Courses$$

$$Y = 2^{Ingredients}$$

$$Z = Dishes$$

You

http://dietexpertnj.wordpress.com

You

$$favorite_{sid}$$
$$favorite_{mike}$$
$$favorite_{vikram}$$

…

An example

$$favorite_{sid}(dessert) = mario$$

An example

$$favorite_{sid}(dessert) = mario$$

$$mario(\{eggs, mascarpone\}) = tiramisu\_a\_la\_mario$$

An example

$$\mathit{favorite}_{\mathit{sid}}$$

$$(\mathit{dessert})$$

$$(\{\mathit{eggs}, \mathit{mascarpone}\})$$

$$= \mathit{tiramisu\_a\_la\_mario}$$

A slightly different function

- Given ingredients, and a required course, one person is sent out to order a dish from their favorite chef
- Let's say Sid is chosen to order dessert, and the fridge contains eggs and mascarpone

$$order_{sid}(dessert, \{eggs, mascarpone\})$$

Formally...

$$order\colon X \times Y \rightarrow Z$$

$$X = Courses$$

$$Y = 2^{Ingredients}$$

$$Z = Dishes$$

Formally...

$$order \in [X \times Y \to Z]$$

$$X = Courses$$

$$Y = 2^{Ingredients}$$

$$Z = Dishes$$

Setting up a bijection between $[X \to [Y \to Z]]$ and $[X \times Y \to Z]$

- *Intuition:* assuming the choice of chef depends only on the course to be cooked, the result should be the same if Sid is first allotted a course and then provided ingredients, or is sent out to order the same course with the same ingredients

Setting up a bijection between $[X \to [Y \to Z]]$ and $[X \times Y \to Z]$

- *Intuition:* assuming the choice of chef depends only on the course to be cooked, the result should be the same if Sid is first allotted a course and then provided ingredients, or is sent out to order the same course with the same ingredients
- So a plausible bijection is

$$b(\mathit{favorite}_x) \longmapsto \mathit{order}_x$$

where $\mathit{order}_x(c, i) \longmapsto \mathit{favorite}_x(c)(i)$ for all courses c, ingredient sets i

Setting up a bijection between $[X \to [Y \to Z]]$ and $[X \times Y \to Z]$

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caution: there can be other bijections for specific X, Y, Z

$$b(favorite_x) \longmapsto order_x$$

where $order_x(c, i) \longmapsto favorite_x(c)(i)$ for all courses c, ingredient sets i

Surjection and right inverse

- Right inverse of $f : A \longrightarrow B$ is a function $h : B \longrightarrow A$ such that $f(h(b)) = b$ for all $b \in B$
- A function f is surjective iff it has a right inverse

Proof worked out on board here, also see
http://www.cs.cornell.edu/courses/cs2800/2014fa/handouts/jonpak_function_notes.pdf

Quantifiers

- All x has property $F(x)$: $\forall x, F(x)$
- There is some x with property $F(x)$: $\exists x, F(x)$

Space for boardwork, also see
http://www.cs.cornell.edu/courses/cs2800/2014fa/handouts/hubbard_quantifier_notes.pdf

Negating Quantifiers

- It is not the case that all x has property $F(x)$
  $\Leftrightarrow$ there is some x without property $F(x)$

$$\neg(\forall x, F(x)) \Leftrightarrow \exists x, \neg F(x)$$

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Negating Quantifiers

- It is not the case that there is some x with property $F(x)$ $\Leftrightarrow$ all x lack property $F(x)$

$$\neg(\exists x, F(x)) \;\Leftrightarrow\; \forall x, \neg F(x)$$

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Thought for the Day #2

Is this statement true or false?

"All eleven-legged alligators have orange and blue spots"

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