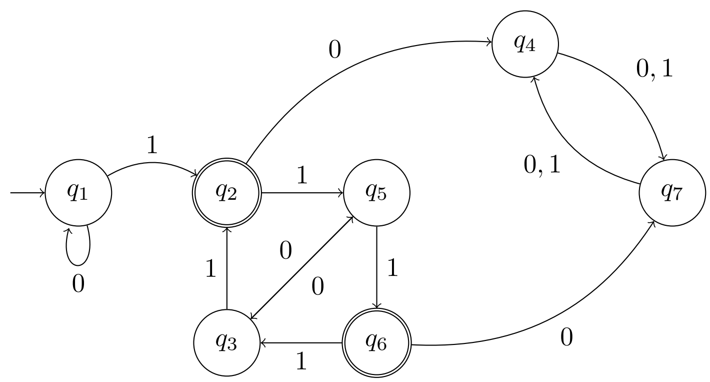
1. There can be different automata that accept the same language. In this problem you will walk through an algorithm that converts a given DFA to an equivalent DFA with the minimal number of states. The key idea is that we want to consolidate "duplicate" states; i.e. states that "behave the same".

   Some of these questions refer to the following machine $M_{example}$:



   (a) We need to consider the language of a machine starting from a given state. Let $L(M, q)$ be the set of strings that M would accept if it started in state q. Write down a formal definition for $L(M, q)$ in terms of the extended transition function.

   (b) We can define an equivalence relation $\sim$ to capture the idea that two states "behave the same": we say that $q_1 \sim q_2$ if $L(M, q_1) = L(M, q_2)$. Prove that $\sim$ is an equivalence relation.

   (c) We now want to create a new machine M' such that $L(M') = L(M)$. The set of states of M' will be the quotient of the states of M by the equivalence relation $\sim$ (in other words, $Q' = Q/\sim$). Explicitly write down the set Q' for the machine $M_{example}$ depicted above. (*Hint: there are three elements of Q', each of which is a set*).

   (d) We can define the transition function δ' for M' as follows: given an equivalence class $q' \in Q'$, we can choose a representative element q of q' and let $\delta'(q', a)$ be the equivalence class of $\delta(q, a)$.

   This may be ambiguous, because there may be many different representatives of the equivalence class q'. Show that the function is in fact well defined, by showing that if we make two different choices of q we still get the same value $\delta'(q', a)$.

   (e) Write down a general expression for the initial state and the set of final states of M'.

   (f) Draw M' for $M_{example}$.