Note: To show that the language of a DFA (or NFA) M is L, you must show that for all $x \in \Sigma^*$, M accepts x **if and only if** $x \in L$. Alternatively, you can show that if $x \in L$ then M accepts x, and if $x \notin L$ then M does not accept L (this is equivalent).

1. (a) Draw the state/transition diagram for a DFA with a binary alphabet which recognizes the set of strings in which every sequence of three successive bits contains a 0. **Note:** the sequences of three successive characters in the string $a_1a_2a_3a_4a_5\cdots$, are $a_1a_2a_3$, $a_2a_3a_4$, $a_3a_4a_5$ etc.

   (b) Explicitly write the five components of this DFA.

2. (a) Let language L be recognized by a DFA with n states. Prove that the language $\Sigma^* \setminus L$ (the complement of L, containing all strings not in L) is also recognized by some DFA with n states.

   (b) Does your construction from (a) work for NFAs as well? If not, give a counterexample.

   (c) Prove that if a language L is recognized by an NFA, then $\Sigma^* \setminus L$ is recognizable by an NFA.

3. Let language L be recognized by a DFA, and b be an arbitrary symbol of its alphabet. Prove that the following language is also recognized by a DFA:

   $$\text{Spaced}(L) = \{a_1ba_2ba_3b\dots ba_n \mid a_1a_2a_3\dots a_n \in L\}$$

   Note: Strings with just one symbol remain unchanged after "spacing".

4. The *factorial* of a natural number n, written $n!$, is the number $1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Prove that the language $L = \{a^{n!} \mid n \in \mathbb{N}\}$ is not recognized by any finite automaton (Hint: prove and use the fact that for any n, there exists some k such that $k! + n < (k+1)!$).