Discrete Structures	Homework 4
CS2800 Fall 2014	Functions

1. State the negation of each of these statements in simple English. (Do not simply prepend a phrase like "It is not the case that...").
   (a) All dogs have fleas.
   (b) There is a horse that can add.
   (c) Every koala can climb.
   (d) There exists a pig that can swim and catch fish.
   (e) In every country, there is a city by a river.
   (f) Old MacDonald had a farm, and on that farm he had a cow.
   (g) Every person in this class understands discrete mathematics.
   (h) Some students in this class do not like discrete mathematics.
   (i) In every mathematics class there is some student who falls asleep during lectures.
   (j) There is a student in this class that can beat every other student in the class at chess.

2. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   (a) Everybody can fool Fred.
   (b) Evelyn can fool everybody.
   (c) Everybody can fool somebody.
   (d) There is no one who can fool everybody.
   (e) Everyone can be fooled by somebody.
   (f) No one can fool both Fred and Jerry.
   (g) Nancy can fool exactly two people.

3. (a) A function $f : A \to A$ is called involutive if for all $x \in A$, $f(f(x)) = x$. Prove or disprove:
      i. if f is involutive, then it is injective.
      ii. if f is involutive, then it is surjective.
   (b) A function $f : A \to A$ is called idempotent if for all $x \in A$, $f(f(x)) = f(x)$. Prove or disprove:
      i. if f is idempotent, then it is injective.
      ii. if f is idempotent, then it is surjective.
   (c) If $f : B \to C$ and $g : A \to B$ are functions, then $f \circ g$ is the function from A to C defined by: $(f \circ g) : x \mapsto f(g(x))$. Prove or disprove: if f and $f \circ g$ are one-to-one, then g is one-to-one.

4. In the first few homeworks, we asserted various facts about sets. We will now prove two of them. Given two sets $A \subseteq S$ and $B \subseteq S$, show that
   (a) $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$.
   (b) $(A \setminus B) \cap (A \cap B) = \emptyset$.

   Note that the formal definition of equality for sets is that $A = B$ if $A \subseteq B$ and $B \subseteq A$, and the formal definition of subset is $A \subseteq B$ if for all $x \in A$, $x \in B$. Definitions for $\cup$, $\cap$, $\setminus$ and $\emptyset$ are on the lecture slides.