Discrete Structures	Homework 1
CS2800 Fall 2014	

1. **(3 points) A broken proof:** A student is asked to prove that $17 = 35$. They submit the following "proof":

$$\begin{aligned}
17 &= 35 \\
17 - 26 &= 35 - 26 && \text{(subtract 26 from both sides)} \\
(17-26)^2 &= (35-26)^2 && \text{(square both sides)} \\
17^2 - 2 \times 26 \times 17 + 26^2 &= 35^2 - 2 \times 26 \times 35 + 26^2 && \text{(expand)} \\
17^2 - 2 \times 26 \times 17 &= 35^2 - 2 \times 26 \times 35 && \text{(cancel } 26^2\text{)} \\
17^2 &= 35^2 - 2 \times 26 \times 18 && \text{(add } 2 \times 26 \times 17 \text{ to both sides)} \\
289 &= 1225 - 936 && \text{(plug it into a calculator)} \\
289 &= 289 && \text{(Q.E.D.)}
\end{aligned}$$

   Explain the faulty reasoning in this "proof".

2. Prove from first principles (set theory, Kolmogorov's axioms) or give a counterexample for each of the following:

   a) **(3 points)** $P(A \cap B) \leq P(A)$ for any events A and B.

   b) **(3 points)** If $A \subseteq B$ but $A \neq B$ then $P(A) < P(B)$.

   c) **(3 points)** If $A_1, A_2, \ldots, A_n$ are mutually exclusive events and $\bigcup_i A_i = S$ (the entire sample space), then for some i, $P(A_i) \geq 1/n$.

3. **(3 points)** Two Cornell students missed their final exam because they were partying in New York City the night before. Desperate for a make-up test, they lied to the professor that they had a flat tire while returning. The professor agreed to give them a make-up test, as long as the students sat in separate rooms. When they opened the paper, they found a single question, worth 100 points: "Which tire was it?"

   What's the probability the two students will give the same answer? Justify your result and clearly state any assumptions you made.

4. **Drawing pairs:**

   a) **(2 points)** You have 10 red, 10 green and 10 blue pairs of socks in a drawer. What's the probability that if you randomly pull out two socks without looking, they will be the same color?

   b) **(3 points)** You have 10 red, 10 green and 10 blue pairs of shoes in a (very large) drawer. What's the probability that if you randomly pull out two shoes without looking, they will be the same color *and* a left-right pair?

   Justify your answer in both cases.