Let $f : A \to B$ be one-to-one and $g : A \to B$ be onto. Since g is onto, for every $b \in B$ there exists at least one inverse image of b in A. Choose any such element and call it a_b ($g(a_b) = b, a_b \in A$). Since the function h, defined as $h : B \to A, h(b) = a_b$ is a function (guaranteed by ontoness of g), and it is one-to-one by construction, we may use f and h to prove that there exists a one-to-one and onto mapping from A to B as done in the class.

For $a \in A$ define a sequence as follows:

$..., f^{-1}(h^{-1}(a)), h^{-1}(a), a, f(a), h(f(a)), ...$

which consists of alternating elements of A and B.

Observe that this sequence may terminate to the left, since f^{-1} or h^{-1} might not be defined at that point. Also observe that this sequence may be cyclic, if elements are recurred, or doubly infinite.

Note that if the sequence starts to repeat elements to the right, it should also start to repeat the same elements to the left, stopping to the left or infinitely continuing is not an option since f and h are one-to-one and that would require two elements to have same image, violating one-to-oneness.

Therefore, all possible options for the sequence are, being cyclic, doubly infinite, or one sided infinite. Also f and h being one-to-one guarantees that every element is in exactly one such sequence, because the same argument could be used to show that the elements to the right and left of it are the same in two sequences. So all such sequences make a partition of A and B.

Define a one-to-one onto function from A to B as follows:

- For cyclic and doubly infinite sequences, use f or h to map elements of the sequence in A to those in B.

- For one-sided infinite sequences, map elements using f if the leftmost element is in A or using h if it is in B.