1 Strings

- ϵ denotes the *empty string*.
- 0^i is the string 0 repeated i times. So $0^3 = 000$, $(01)^5 = 0101010101$, and $(0001011)^0 = \epsilon$.
- 0^* is the string 0 repeated 0 or more times $(\epsilon, 0, 00, 000, 0000, ...)$.
- The " + " operator for strings means *or* (you can choose either of the strings surrounding the +). So $(0+1)(1+0)$ is the set containing the strings $00, 01, 10$, and 11. $(0+1+2)(0+1)$ is the set containing the strings $00, 01, 10, 11, 20$, and 21. And $(3+4)^*$ covers all strings with any combination of characters that are either 3 or 4 (e.g. 43444433).

2 Sets

- Brackets {} define *sets*. The members of a set are separated by commas. $S = \{a, b, c, d\}$ defines S as a set that contains the elements a, b, c, and d.
- $\in$ is the symbol for *set membership*. In the previous example where $S = \{a, b, c, d\}$, we say $d \in S$, or d is an element of S.
- $\emptyset$ is the *empty set*. It is the set that contains no elements.
- $S \subseteq T$ means that S is a *subset* of T. If $S \subseteq T$, then if x is an element of S, then x must also be an element of T. $\{1,2,3\} \subseteq \{1,2,3\}$, and $\{1,2,5\} \subseteq \{1,2,3,4,5\}$, but $\{1,2,4\} \nsubseteq \{1,2,3\}$.
- $S \subset T$ means that S is a *proper subset* of T. If $S \subset T$, then if x is an element of S, then x must also be an element of T and there exists at least one element in T that is not in S. $\{1,2,5\} \subset \{1,2,3,4,5\}$, but $\{1,2,3\} \not\subset \{1,2,3\}$ and $\{1,2,4\} \not\subset \{1,2,3\}$.
- 2^S (or $\mathcal{P}(S)$) is the *power set* of S, or the set of all subsets of S. So
$$2^{(\{1,2,3\})} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$
- $\bar{S}$ is the *complement* of S, or the set containing all elements that are not members of S. When taking the complement of a set, there is assumed to be some universal set around which the complement is taken. So, if the universal set is the set of all positive integers, and S is the set of all primes, $\bar{S}$ is the set of all composite numbers.
- $A \setminus B$ is the *set-theoretic difference* of A and B. This is defined as all of the elements that are members of A, but not members of B. If $A = \{1,2,3\}$ and $B = \{1,3,4,7\}$, then $A \setminus B = \{2,3\}$ and $B \setminus A = \{4,7\}$.
- $A \cup B$ is the *union* of A and B. This is defined as the set of all elements that are in either A or B. If $A = \{1,3,5,8\}$ and $B = \{1,2,3,4,5,6,7\}$, then $A \cup B = \{1,2,3,4,5,6,7,8\}$.
- $A \cap B$ is the *intersection* of A and B. This is defined as the set of all elements that are members of both A and B. If $A = \{1,3,5,8\}$ and $B = \{1,2,3,4,5,6,7\}$, then $A \cap B = \{1,3,5\}$.
- You can also take unions or intersections over many sets at a time:
$$\bigcup_{i=1}^{n} S_i = S_1 \cup S_2 \cup S_3 \cup ... \cup S_n$$
and
$$\bigcap_{i=1}^{n} S_i = S_1 \cap S_2 \cap S_3 \cap ... \cap S_n.$$

- S^* is the *Kleene closure* of S, or the set of all strings that can be made of elements of S. This closure allows empty strings, so ϵ is in the closure of any set. So $\{1\}^* = \{\epsilon, 1, 11, 111, 1111, ...\}$. Another way of writing this would be $\{1^i | i \in N\}$. We could also make a slightly more complicated set, $\{00, 10\}^* = \{(00 + 10)^i | i \in N\}$. Some strings in $\{00, 10\}^*$ are $\epsilon, 00, 10, 0000, 0010, 1000$, and 1010.
- $A \times B$ is the *Cartesian product* of A and B. This is the set of all pairs (a, b) such that $a \in A$ and $b \in B$. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, then
$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}.$$
- $A \cdot B$ is set concatenation. It behaves like a Cartesian product, except that instead of using pairs (a, b), we concatenate a and b. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, then $A \cdot B = \{1a, 1b, 1c, 2a, 2b, 2c, 3a, 3b, 3c\}$.
- $|A|$ is the *cardinality* of A, or the number of elements in A. If $A = \{1, 2, 4\}$, then $|A| = 3$.

3 Problems or Concerns

This document was intended to define a consistent notation that will reflect the notation that was used in lecture and will be used on assignments and prelims. If there are is any notation that you encounter that is not on this sheet, or if there is any notation on this sheet that contradicts what you have been taught in lecture or see on the assignments, please email cs2800staff-l@cs.cornell.edu so we can correct the problem.