CS 2800: Discrete Structures

Homework 5

Due Monday, October 1, 2012

Please write your netid on the upper right corner of all pages. Grading for all problems will be based on neatness, style, and correctness.

1. Consider the set of all two by two matrices of real numbers and the operation of multiplication given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

   (a) Does the set of all 2×2 matrices whose elements are real numbers form a group under multiplication?

   (b) A matrix A is nonsingular if there exists a matrix A^{-1} called the inverse of A such that $A \times A^{-1} = I$ where I is the identity matrix with ones on the diagonal and all other elements zero. Does the set of all nonsingular 2×2 matrices whose elements are real numbers form a group under multiplication? Is it commutative?

2. **(Halting problem)** Suppose you could write a computer program that would take as input another computer program P and an input I to it. Your computer program would halt on all inputs (P, I) and correctly state whether the program P that was input would halt on input I. What contradiction would this lead to? Hint: Could you compute something that is not computable?

3. Find close form solutions for the following recurrence equations. Check the first four values of your answer.

   (a) $f(n) = 8f(n-1) - 15f(n-2)$ with boundary conditions $f(0) = 0$ and $f(1) = 1$

   (b) $f(n) = 4f(n-1) - 3f(n-2)$ with boundary conditions $f(0) = 8$ and $f(1) = 7$

4. (a) $f(n) = 4f(n-2)$ with boundary conditions $f(0) = 1$ and $f(1) = 0$

   (b) $f(n) = f(n-1) + 8f(n-2) - 12f(n-3$ with boundary conditions $f(0) = 0$, $f(1) = 1$, and $f(2) = 2$

5. Solve the recurrence equation $f(n) = -4f(n-2)$ for which the characteristic equation has imaginary roots. Use the boundary conditions $f(0) = 0$ and $f(1) = 1$. Clearly $f(n)$ must be real for all values of n.

   (a) What is the characteristic equation?

   (b) What are the roots?

   (c) What is the most general form of solution to the recurrence equation?

   (d) Use the boundary conditions to find the specific solution of the recurrence equation and the boundary conditions.

   (e) What values does your solution give for $f(n)$ for $n = 2, 3, 4,$ and 5?

6. What is the form of the most general solution to the recurrence equation $f(n) = 5f(n-1) - 8f(n-2) + 4f(n-3)$?