CS 2800: Discrete Structures

Homework 4

Due Monday, September 24, 2012

Please write, in large print, your netID on the upper right corner of all pages (failure to do so will result in a point deduction). Grading for all problems will be based on neatness, style, and correctness. As always, explain all answers.

Question 1

Prove the following statement using induction:

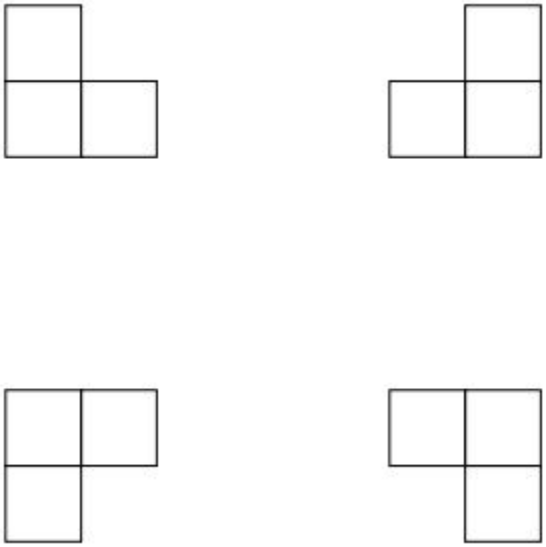
$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Question 2

Prove the following:

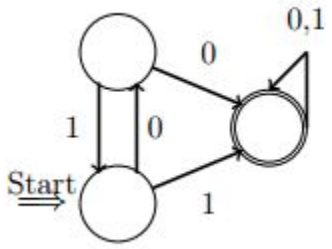
1. Any $2^n \times 2^n$ checker board that is missing the tile from the upper left hand corner can be covered with the tiles from the set below.
2. Any $2^n \times 2^n$ checker board that is missing a tile from *anywhere* in the board can be covered with the tiles from the set below.

In each case, clearly state your inductive hypothesis.



Question 3

Prove, by induction, that the automaton below accepts the set of all strings of 0s and 1s that either start with a 1 or have two consecutive 0s or two consecutive 1s.



If you find that your inductive proof includes a number of similar cases to prove, feel free to clearly define just two of these cases and omit the others, assuming that we understand by their structure what you aimed to write for those cases that are omitted.

Question 4

Prove that the game Towers of Hanoi can be solved in $2^n - 1$ moves. The game consists of three rods and a number of disks of different sizes which can slide onto any rod. The puzzle starts with the disks in a stack (smallest disk on top, and each disk below is larger than the one before it) on the leftmost rod. The objective is to move the entire stack to another rod in accordance with the following rules:

1. Only one disk may be moved at a time.
2. Each move consists of taking the upper disk from one of the rods and siding it onto another rod.
3. No disk may be placed on top of a smaller disk.

For further clarification, please refer to this link: `http://www.mazeworks.com/hanoi/`

Question 5 (Bonus Question*)

Give a zero knowledge proof that a graph has a Hamilton circuit. Hint: Encrypt things so the other person can ask for either the graph or the Hamilton circuit.

Question 6

Let S = $\{a^i b a^{2i} b | i >= 0\}$. Let D = {bS* $\cap$ babS^*a*b}. Is D equal to $\emptyset$? If yes, explain why that is. If no, describe the complete set of strings of a's and b's that are included in the set. Note: By bS* we mean $\{b\} \cdot$ S*. We have dropped the brackets for convenience.

* Homework 4 will be graded out of 50. Bonus question has extra points to the overall homework grade.