CS 2800: Discrete Structures

Homework 3

Due Monday, September 17, 2012

[Version 2 - Updated Tuesday, September 11, 2011]

Please write your netid on the upper right corner of all pages. Grading for all problems will be based on neatness, style, and correctness.

1. Consider the non negative integers $N = \{0, 1, 2, \ldots\}$, and define $i \equiv j$ to be whether the remainder of i divided by 5 equals the remainder of j divided by 5.

   (a) Is the $\equiv$ relation reflexive? Is it symmetric? Is it transitive?

   (b) What are the equivalence classes?

   (c) Select the smallest integer in each equivalence class to represent the equivalence class. Let $\{i\}_{mod}$ be the representative for the class containing i. Prove the following statements:
   i) $\{i\}_{mod} + \{j\}_{mod} \equiv \{i + j\}_{mod}$
   ii) $\{i\}_{mod} \times \{j\}_{mod} \equiv \{i \times j\}_{mod}$.

   (d) Which of the following are valid statements? Explain.

      i. $5 \equiv 10$

      ii. $6 \equiv 7$

   (e) Write out the addition and multiplication tables for the representative elements of the equivalence classes so that $\{i\}_{mod} + \{j\}_{mod} \equiv \{i+j\}_{mod}$ and $\{i\}_{mod} \times \{j\}_{mod} \equiv \{i \times j\}_{mod}$

   (f) How can one add negative numbers to this system? That is, replace N with Z.

2. (a) List three groups besides $(Z, +)$.

   (b) Give an example of a non commutative group.

   (c) How do you prove that $\{I, r, r^2, r^3, f, fr, fr^2, fr^3\}$ is a group under concatenation?

3. Prove that between every two rationals there exists a real.

4. Prove that between every two reals there exists a rational.

5. Let R be the reals. Is there a one-to-one mapping from R^2 to R?

6. A finite game is a game with a finite number of steps and no ties. Does every finite game have a winning strategy for either player 1 or player 2? Justify your answer.