CS 2800: Discrete Structures

Homework 2

Due Monday, September 10, 2012

Please write your netid on the upper right corner of all pages. Grading for all problems will be based on neatness, style, and correctness.

1. Which of the following classes of sets are closed under each of the following operations: union, intersection, power set operation? Explain your answer.
   (a) the class of all finite sets
   (b) the class of all infinite sets
   (c) the class of all sets of even cardinality
   (d) the class of all sets of odd cardinality

2. Prove that $S^* = \bigcup_{i=0}^{\infty} S^i$ is the smallest set containing $S \cup \{\epsilon\}$ and closed under concatenation.

3. Is there a one-to-one mapping from the set of all finite length strings over the alphabet $\{a, b, \ldots, z\}$ to the set of all finite length strings over the alphabet $\{0, 1\}$? Please explain your reasoning.

4. If there is a one-to-one mapping from A to B, and also an onto mapping from A to B, does this imply a one-to-one onto mapping from A to B? Please justify your answer.

5. Classify the following sets as countably infinite or not countably infinite. Please give a brief explanation with your answers.
   (a) all finite subsets of integers
   (b) all subsets of integers
   (c) all finite subsets of reals
   (d) the set of all finite length strings
   (e) all computer programs

6. Let f be an one-to-one mapping from S to T.
   (a) Give an example of two sets S and T and a mapping f where the fact that S and T are isomorphic implies that f is also onto.
   (b) Give an example of two sets S and T and a mapping f where the fact that S and T are isomorphic does not imply that f is also onto.