CS 2800: Discrete Structures

Homework 11

Due Monday, November 19, 2012

Please write your netid in large font on the upper right corner of all pages. Grading for all problems will be based on neatness, style, and correctness.

1. Let A and B be independent events.
   (a) Prove that A and $\overline{B}$ are independent events.
   (b) Prove that $\overline{A}$ and $\overline{B}$ are independent events.
2. Give an example showing that the square of the expected value of a random variable is not necessarily equal to the expected value of the variable squared.
3. Suppose that 8% of players use steroids, that a player tested for steroids tests positive 96% of the time, and that a player not on steroids tests positive 9% of the time. What is the probability that a player who tests positive actually takes steroids?
4. How accurate would a test need to be to detect a rare disease that occurs in only 10^{-6} of the population? Say you want an individual to have the disease with probability 0.99 if the test says yes. Assume that as you improve the detection of the disease in people who have it you decrease the false positives for people who do not have the disease. That is Prob(no|disease) = Prob(yes|no disease). You only need to give an order of magnitude accuracy. Thus, you can replace $1 - 10^{-6}$ by 1 in any formula in which it occurs. Suggestion: Use the letter a for the unknown rather then x so you do not confuse the variable with the multiplication symbol.
5. (a) Consider a set of jars with marbles in them. As the number of jars increases to infinity we observe that the expected number of marbles per jar also goes to infinity. If we select a jar at random, what can we say about the probability that the jar will have at least one marble? Give a brief explanation for your answer.
   (b) If the expected number of marbles in a jar had gone to zero, what could we have said about the probability that a jar selected at random would have at least one marble? Again given a brief explanation for your answer.