

CS 2800 - Homework 9 - Due Wednesday April 21  
at the beginning of lecture

**INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK**

**NETID:**

**NAME:**

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problem	grade	memo
1		
2		
3		
4		
5		
total		

You should justify/prove all your answers.

### Problem 1

Show that the following are equivalent:

- (a) A graph  $G$  with  $n$  nodes is connected and has no cycles.
- (b) A graph  $G$  with  $n$  nodes is connected and has  $n - 1$  edges.

### Problem 2

Suppose  $G$  is a graph with  $n$  nodes and  $m$  edges.

- (a) What is the minimum number of connected components  $G$  could have?
- (b) What is the maximum number of connected components  $G$  could have?

### Problem 3

In this problem we will consider the  $n$  dimensional cube. For all  $n \geq 1$ , the graph  $\text{Cube}_n$  is defined as follows. The vertex set is all possible 0/1 strings of length  $n$  and  $\{u, v\}$  is an edge of  $\text{Cube}_n$  iff  $u$  and  $v$  differ in exactly one position. For  $n = 1$  we have a single edge, for  $n = 2$  we have a square, for  $n = 3$  we have the edges of a cube.

Find the chromatic number  $\chi(\text{Cube}_n)$ .

### Problem 4

Consider the  $n$  dimensional cubes defined in the last problem. For which  $n$  is  $\text{Cube}_n$  Eulerian?

### Problem 5

We can define the notion of a random graph on  $n$  nodes  $V = \{1, \dots, n\}$  as follows. For each possible edge  $\{i, j\}$  we include the edge in the graph with probability  $p$  and don't include it with probability  $1 - p$ .

- (a) What is the expected number of edges in a random graph?
- (b) What is the expected number of triangles? (a triangle is a cycle of length 3)
- (c) What is the probability that a random graph is complete?