CS 2800 - Homework 7 - Due Wednesday April 7 at the beginning of lecture

INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK

NETID:

NAME:

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problem	grade	memo
1		
2		
3		
4		
5		
total		

You should justify/prove all your answers.

Problem 1

Suppose you have a drawer with 10 red socks and 10 blue socks and you pick some socks without looking at them.

(a) How many should you pick to be sure of having at least 2 of the same color?

(b) How many should you pick to be sure of having at least 2 blue socks?

Justify your answers.

Problem 2

How many positive integers strictly less than 2102 are divisible by 2,3 or 5?

Problem 3

Suppose we have a collection of subsets of $\{1, 2, \ldots, n\}$ such that each pair of subsets has at least one element in common. What is the maximum possible size of the collection?

Problem 4

Show that if n is a positive integer, than $\binom{2n}{2} = 2\binom{n}{2} + n^2$

(a) using a combinatorial argument (show both sides count the same thing)

(b) by algebraic manipulation

Problem 5

Count the number of paths that go from the bottom left corner to the top right corner of the $n \times k$ grid where each step of the path moves to the right or up. The figure shows a 4×10 grid with such a path highlighted.