

CS 2800 - Homework 6 - Due Wednesday March 17
at the beginning of lecture

INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK

NETID:

NAME:

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problem	grade	memo
1		
2		
3		
4		
5		
total		

You should justify/prove all your answers.

Problem 1

Consider a statement S of the form $A_1 \wedge \dots \wedge A_k \Rightarrow B$ where the A_i and B are arbitrary (not necessarily primitive) propositions. We can define a formal derivation of S by a sequence of statements C_1, \dots, C_n such that $C_n = B$ and each C_i is either an axiom, one of the “premises” A_i , or a statement derivable from C_1, \dots, C_{i-1} using an inference rule.

Consider the following situation:

I will buy Apple stock only if the iPad is for sale. I will buy Apple stock or go on vacation (perhaps both). I am not going on vacation. Therefore the iPad is for sale.

(a) Translate this into a statement S of the form described above, with appropriately defined primitive propositions.

(b) Give a formal derivation of S using an axiom system with no axioms and the inference rules below. Explain each step of the derivation.

Disjunctive syllogism

Modus ponens

$$\begin{array}{l} A \vee B \\ \neg B \\ \hline A \end{array}$$

$$\begin{array}{l} A \\ A \Rightarrow B \\ \hline B \end{array}$$

Problem 2

Determine if the formula $\exists x \forall y (x \leq y^2)$ is true if the domain for the variables consists of

- (a) the positive real numbers.
- (b) the integers.
- (c) the nonzero real numbers.

In each case prove the statement is true or false.

Problem 3

For each of these arguments give a formal formula in first order logic which captures it. Specify the domain for the variables in each case (one domain per formula). Determine if each argument is correct by checking if the formula is valid.

(a) All students enrolled in cs2800 are happy. Joe is not enrolled in cs2800. Therefore Joe is not happy.

(b) All students like all good classes. cs2800 is a good class. Therefore some student likes cs2800.

Problem 4

How many strings of four decimal digits (a decimal digit is a number between 0 and 9)

- (a) do not contain the same digit twice?
- (b) end with an even digit?
- (c) have exactly three digits that are 9s?

Problem 5

An n -bit *boolean* function f maps a 0/1 string of length n to 0 or 1, $f : \{0, 1\}^n \rightarrow \{0, 1\}$. An n -bit boolean function *depends* on bit i if \exists two strings A and B s.t. A and B differ only in position i and $f(A) \neq f(B)$.

- (a) Count the number of n -bit boolean functions.
- (b) Count the number of n -bit boolean functions that do **not** depend on bit i .