CS 2800 - Homework 4 - Due Wednesday Feb. 24 at the beginning of lecture

INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK

NETID:

NAME:

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problem	grade	memo
1		
2		
3		
4		
5		
total		

You should justify/prove all your answers.

Problem 1

Let a, b, c and d be integers.

(a) Show that if $a|c$ and $b|d$ then $ab|cd$.

(b) Prove or disprove that if $a|bc$ then $a|b$ or $a|c$.

Problem 2

Let a, b, c and d be integers and m be an integer greater than one.

Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $a - c \equiv b - d \pmod{m}$

Problem 3

Use the Euclidean algorithm to compute the gcd of 330 and 308. Show your work. Write $\gcd(330, 308)$ as a linear combination of 330 and 308.

Problem 4

Steve Jobs decided he will not release the iPad unless you give him a jug of water with precisely c gallons. After searching all day for a proper container you only found two jugs, one that holds a gallons, and one that holds b gallons.

Prove that you can measure precisily c gallons if and only if c is a multiple of $\gcd(a, b)$.

You should assume that a, b and c are positive integers, and the only operations we can do involve filling a jug to the top, dumping the contents of a jug out, and dumping the contents of one jug into another jug.

Problem 5

Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.

Hint: note that $k^2 + k = k(1 + k)$.