

CS 2800 - Homework 3 - Due Wednesday Feb. 17
at the beginning of lecture

INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK

NETID:

NAME:

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problem	grade	memo
1		
2		
3		
4		
5		
6		
total		

You should justify/prove all your answers.

Problem 1

Determine whether $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ is onto and/or one-to-one if

- (a) $f(m, n) = 2m - n$.
- (b) $f(m, n) = m^2 - n^2$.
- (c) $f(m, n) = m + n + 1$.

Problem 2

Let S be a subset of a universal set U . The *characteristic function* f_S of S is the function from U to $\{0, 1\}$ such that $f_S(x) = 1$ if $x \in S$ and $f_S(x) = 0$ otherwise.

Show the following holds for all x

- (a) $f_{A \cap B}(x) = f_A(x)f_B(x)$.
- (b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x)f_B(x)$.
- (c) $f_{\bar{A}}(x) = 1 - f_A(x)$.
- (d) $f_{A - B}(x) = f_A(x)(1 - f_B(x))$.

Problem 3

Show that these statements about the real number x are equivalent:

- (a) x is rational
- (b) $x/2$ is rational

Problem 4

The value of the *Euler ϕ function* at the positive integer n is defined to be the number of positive integers less than or equal to n that are relatively prime to n .

- (a) Find $\phi(4)$, $\phi(10)$, $\phi(13)$.
- (b) Show that n is prime if and only if $\phi(n) = n - 1$.

Problem 5

Use induction to prove that $n! < n^n$ for all integers $n \geq 2$.

Problem 6

We are given a chocolate bar with $m \times n$ squares of chocolate, and our task is to divide it into mn individual squares. We are only allowed to split one piece of chocolate at a time using a vertical or horizontal break.

For example, suppose the chocolate bar is 2×2 . The first split makes two pieces, both 2×1 . Each of these pieces requires one more split to form single squares. This gives a total of three splits.

Use strong induction to show that dividing a chocolate bar with $m \times n$ squares takes at most $mn - 1$ splits.

Hint: Induct on the size of a bar, where the size of an $m \times n$ bar is mn .