

# CS 2800 - Homework 2 - Due Wednesday Feb. 10 at the beginning of lecture

## Problem 1

Find two sets  $A$  and  $B$  such that  $A \in B$  and  $A \subseteq B$ .

## Problem 2

Let  $A$  and  $B$  be sets of elements from a universe  $X$ .

- (a) Show that  $\overline{A \cup B} = \overline{A} \cap \overline{B}$ .
- (b) Show that  $A - B = A \cap \overline{B}$ .

## Problem 3

Let  $f$  be a function from  $\mathbb{Z}$  to  $\mathbb{Z} \times \mathbb{Z}$ . What is the domain, image and range of  $f$  defined by

- (a)  $f(x) = (1, 0)$  if  $x \geq 0$  and  $f(x) = (0, 1)$  if  $x < 0$ .
- (b)  $f(x) = (a, b)$  such that  $a$  is the quotient of  $|x|$  divided by 5 and  $b$  is the remainder.

IMPORTANT: Rosen (page 134) uses different terminology than in class:

Rosen	Class/Homework
Domain	Domain
Codomain	Range
Range	Image

## Problem 4

Which of the following relations on the integers are equivalence relations? Justify your answers.

- (a)  $R$  such that  $(a, b) \in R$  if  $|a - b| \leq 2$
- (b)  $R$  such that  $(a, b) \in R$  if  $|a - b| \leq 0$
- (c)  $R$  such that  $(a, b) \in R$  if  $|a| = |b|$
- (d)  $R$  such that  $(a, b) \in R$  if  $|a| = -|b|$

### **Problem 5**

Suppose  $f$  is a function from  $A$  to  $B$  where  $A$  and  $B$  are finite sets with  $|A| = |B|$ . Show that  $f$  is one-to-one if and only if it is onto.

### **Problem 6**

Use induction to show that  $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ .