CS 2800 - Homework 1 - Due Wednesday Feb. 3

Problem 1

What is the cardinality of each of these sets

(a) $\{a\}$
(b) $\{a, \{a\}\}$
(c) $\{\{a, a\}\}$
(d) $\{\{a\}\}$
(e) $\{a, \{a\}, \{a, \{a\}\}\}$

Problem 2

Let A be a set with m elements and B be a set with n elements. What is the cardinality of $A \times B$? Justify your answer.

Problem 3

Russel's paradox: Let S be the set that contains a set x if x does not belong to itself,

$$S = \{x | x \notin x\}.$$

(a) Show that assuming S is a member of itself leads to a contradiction.
(b) Show that assuming S is not a member of itself leads to a contradiction.
This paradox can be avoided by restricting the types of elements that sets can have.

Problem 4

Let A, B and C be sets

(a) Show that $B \cap (A - B) = \{\}$.
(b) Show that $A - (B \cup C) = (A - B) \cap (A - C)$.

Problem 5

Give the transitive closure of the following relations:

(a) $\{(1, 3), (1, 4), (2, 4), (3, 1), (3, 2)\}$.
(b) The $\leq$ relation - consisting of pairs of integers (a, b) such that $a \leq b$.
(c) The $+1$ relation, consisting of pairs $(n, n + 1)$ where n is a natural number.

Problem 6

Given an example of a function from $\mathbb{Z}$ to $\mathbb{Z}$ that is

(a) one-to-one but not onto

(b) onto but not one-to-one

(c) both onto and one-to-one - but not the identity

(d) neither one-to-one nor onto

Justify your answers.

Problem 7

A partition of a set A is a collection of nonempty sets P such that the union of all sets in P is A, and the intersection of a pair of different sets in P is empty.

Let R be a relation on $\mathbb{N}$ such that $(x, y) \in R$ if there exists $z \in \mathbb{Z}$ such that $x = y + 3z$.

(a) Show R is an equivalence relation.

(b) What are the equivalence classes of R?

(c) Show these equivalence classes form a partition of $\mathbb{N}$.