CS 280 – Fall 2007
Review Problems for the Final

1. Convert the following statements into first-order logic and determine if the given argument is sound:

   Rainy days make gardens grow.
   Gardens don't grow if it is not hot.
   It always rains on a day that is not hot.
   Therefore, if it is not hot, then it is hot.

2. What is the cardinality of each of the following sets?

   (a) $\emptyset$

   (b) $\{\emptyset\}$

   (c) $\{\emptyset, \{\emptyset\}\}$

   (d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

3. We define the set $S = \{x | x \notin S\}$. Does $S \in S$? Does $S \notin S$?

4. Show that if A and B are countable sets, then $A \times B$ is also countable.

5. What is the best big-O function for

   (a) $n^3 + \sin n^7$

   (b) $(x+2)\log_2(x^2+1) + \log_2(x^3+1)$

6. What is the time complexity of the following algorithms?

   (a) One that prints out all the ways to place the numbers $1, 2, \ldots, n$ in a row.

   (b) One that enumerates all the subsets of a given finite set of size n.

7. Prove that if n is an integer that is not a multiple of 3, then $n^2 \equiv 1 \pmod 3$.

8. Show using induction that $11 | (10^{2k+1} + 1)$ for all non-negative integers k.

9. Show using induction that for all positive integers n:

$$1 - 2 + 2^2 - 2^3 + \ldots + (-1)^n 2^n = \frac{2^{n+1}(-1)^n + 1}{3}$$

10. How many strings of eight English letters are there

    (a) if letters can be repeated?

    (b) if no letter can be repeated?

(c) that start with X, if letters can be repeated?

(d) that start with X, if no letter can be repeated?

(e) that start and end with X, if letters can be repeated?

(f) that start with the letters BO (in that order), if letters can be repeated?

(g) that start and end with the letters BO (in that order), if letters can be repeated?

(h) that start or end with the letters BO (in that order), if letters can be repeated?

11. Show that there are at least 6 people in California (population 36 million) with the same three initials who were born on the same day of the year (but not necessarily the same year). Assume everyone has three initials.

12. How many different strings can be made from the letters in $ORONO$ using some or all the letters?

13. What is the probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

14. A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third of the time and a 0 the rest of the time. When a 0 is sent, the probability that it is received correctly is 0.9 and the probability that the 0 is corrupted into a 1 is 0.1. When a 1 is sent, the probability that it is not corrupted is 0.8 and the probability that it is is 0.2.

    (a) Find the probability of receiving a 0.

    (b) Find the probability that a 0 was transmitted given that a 0 was received.