

The course will discuss mathematical techniques and models over a large range of computer science applications, touching on issues in systems, databases, networking, artificial intelligence, cryptography, Web search, and algorithm design. The following outline of topics is meant to convey the course organization and the range of subjects covered in the course; the details and organization are subject to change.

1. Boolean formulas

- Circuits and decision rules.
- Quantifiers.
- Searching a game tree: the internals of computer game-playing programs.

2. Induction

- Basics of proofs by induction.
- Induction paradoxes: sandpiles, surprise quizzes, and the smallest number that cannot be described in fewer than 100 symbols.

3. Numbers

- Divisibility, modular arithmetic, and Euclid's algorithm.
- Prime numbers.
- RSA and public-key cryptography: The problem of communicating securely with someone you've never met.

4. Sets and Counting

- Counting arguments.
- The pigeon-hole principle.
- Applications to data mining: What's market-basket data and why does Wal-Mart have several hundred terabytes of it?

5. Graphs

- Connectivity and trees.
- Distances in graphs, and the six degrees of separation in social networks.
- Graph coloring, and contention resolution in distributed systems.
- The structure of the World Wide Web: communities and giant components.

6. Relations and Partial Orders

- Symmetry, transitivity, and equivalence relations.
- Partial orders, chains, and antichains: Towards a theory of parallel computing.

7. Probability

- Events, independence, and a randomized scheduling algorithm.
- Conditional probability and spam-filtering heuristics.
- Random variables and their expectations: How long do you need to collect baseball cards before you get every one?
- Deviations in random variables: When should you flag anomolous behavior as fraudulent?
- Random graphs: modeling how peer-to-peer file-sharing systems grow.