Examples

- Pond has 50 fish of which 10 are tagged, then 9 fish are netted.

$$p(\text{exactly 2 netted are tagged}) = \frac{\binom{10}{2}\binom{40}{7}}{\binom{50}{9}}$$

- Now the pond has 10 A-fish, 15 B-fish, 20 C-fish, and 5 others. Again net 9 fish.

$$p\left(\text{exactly } \begin{matrix} 2 \text{ A-fish} \\ 3 \text{ B-fish} \\ 2 \text{ C-fish} \end{matrix}\right) = \frac{\binom{10}{2}\binom{15}{3}\binom{20}{2}\binom{5}{2}}{\binom{50}{9}}$$

- Can also build a probability list of events, e.g., values of sum of 2 dice ...

2	1/36	6	5/36	10	3/36
3	2/36	7	6/36	11	2/36
4	3/36	8	5/36	12	1/36
5	4/36	9	4/36		

So the sums are not equi-probable, yet we can use this data to weight our events ...

$$p(\text{get a prime sum}) = p(\text{get } 2, 3, 5, 7, \text{ or } 11)$$
$$= \frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36} = \frac{15}{36}$$

Conditional Probability

If A and B are independent events then $p(A \cap B) = p(A)p(B)$. Now let $p(A \mid B)$ denote the probability of A given B, then with A and B still independent we get

$$p(A \cap B) = p(A \mid B)\, p(B).$$

This inspires a definition useful in dependent situations (provided $p(B) \neq 0$) of

$$p(A \mid B) := \frac{p(A \cap B)}{p(B)}.$$