Characteristics of Distributions

Definition

Let X be a discrete random variable with p.d.f. f on S, and $w(x)$ a 'payoff function' on S. Then the expectation of $w(x)$ is

$$E[w(x)] := \sum_{x \in S} w(x) f(x) \quad \text{if the sum exists.}$$

Notice that

(i) for λ constant, $E[\lambda] = \lambda$.

(ii) for λ constant, $E[\lambda . w(x)] = \lambda . E[w(x)]$.

(iii) $E[w(x) + v(x)] = E[w(x)] + E[v(x)]$.

alias, E is a linear operator

(iv) for x, y stat. indep., $E[w(x) . v(y)] = E[w(x)] . E[v(y)]$.

Definitions

If in the above we let $w(x) = x \ \forall x \in S$, then $E[w(x)] = E[x] = \mu$ gives the mean of S. To measure the spread of a distribution we could use $E[|x-\mu|]$, though this is computationally tricky to manipulate, so we define the variance of S by

$$\text{var}(x) = \sigma^2 := E[(x-\mu)^2]$$

RMS, root mean square

and the standard deviation of S by $\sigma := \sqrt{E[(x-\mu)^2]}$.

Notice that $\sigma^2 = E[(x-\mu)^2] = E[x^2 - 2x\mu + \mu^2]$

$$= E[x^2] - 2\mu E[x] + \mu^2 = E[x^2] - (E[x])^2.$$

Also, if $y = ax + b$, then

$$\mu_y = E[ax+b] = a\mu_x + b,$$

and

$$\sigma_y^2 = E[(ax + b - a\mu_x - b)^2] = E[a^2(x-\mu_x)^2] = a^2 \sigma_x^2$$

$$\Rightarrow \quad \sigma_y = |a| \sigma_x .$$