Definitions

Suppose an experiment has $X = (x_1, x_2)$ as random variable with p.d.f. $f(x_1, x_2)$, then the marginal p.d.f. for x_1 is the p.d.f. for x_1 with no regard for x_2. Notating this by $f_1(x_1)$, we get

$$f_1(x_1) = \sum_{\substack{\text{all } x_2 \text{ with} \\ (x_1, x_2) \in S}} f(x_1, x_2) .$$

Then the marginal distribution for x_1 is $F_1(a_1) = \sum_{x_1 \le a_1} f_1(x_1)$.

For trivariate distributions we can define both

$$f_{12}(x_1, x_2) = \sum_{x_3} f(x_1, x_2, x_3) \quad \text{and} \quad f_1(x_1) = \sum_{x_2} \sum_{x_3} f(x_1, x_2, x_3)$$

as marginal distributions for (x_1, x_2) and x_1 respectively.

E.g. If

$$f(x_1, x_2) = \binom{n}{x_1 \quad x_2 \quad n-x_1-x_2} p_1^{x_1} p_2^{x_2} p_3^{n-x_1-x_2}$$

then

$$\begin{aligned} f_1(x_1) &= \sum_{x_2=0}^{n-x_1} \binom{n}{x_1 \quad x_2 \quad n-x_1-x_2} p_1^{x_1} p_2^{x_2} p_3^{n-x_1-x_2} \\ &= \binom{n}{x_1} p_1^{x_1} \sum_{x_2=0}^{n-x_1} \binom{n-x_1}{x_2} p_2^{x_2} p_3^{n-x_1-x_2} \\ &= \binom{n}{x_1} p_1^{x_1} (p_2 + p_3)^{n-x_1} \\ &= \text{probability for } n \text{ repetitions with probability} = p . \end{aligned}$$

Definition

x_1 and x_2 are statistically independent if

$$f(x_1, x_2) = f_1(x_1) f_2(x_2) \quad \forall (x_1, x_2) \in S .$$

Hence if a given p.d.f. $f(x_1, x_2)$ factors into $\varphi(x_1) \psi(x_2)$ then x_1 and x_2 are stat. indep.. So defining the conditional p.d.f. $f_1(x_1 | x_2) := \dfrac{f(x_1, x_2)}{f_2(x_2)}$ we have x_1 and x_2 statistically independent iff $f_1(x_1 | x_2) = f_1(x_1)$, or sim. with x_2.