Examples (from Stoll, "Set Theory and Logic", 1961)

- No human beings are quadrupeds. All women are human beings. Therefore no women are quadrupeds.

  Let $H(x)$ mean "x is a human being"
  $Q(x)$ " "x is a quadruped"
  $W(x)$ " "x is a woman"

  So we must show that
  $$\begin{array}{l} (\forall x)(H(x) \to \neg Q(x)) \\ (\forall x)(W(x) \to H(x)) \\ \hline (\forall x)(W(x) \to \neg Q(x)) \end{array}$$

  (1) $\forall x (H(x) \to \neg Q(x))$ rule p
  (2) $\forall x (W(x) \to H(x))$ rule p
  (3) $W(y) \to H(y)$ rule ui and (2)
  (4) $H(y) \to \neg Q(y)$ rule ui and (1)
  (5) $W(y) \to \neg Q(y)$ rule t and (3), (4)
  (6) $\forall x (W(x) \to \neg Q(x))$ rule ug and (5)

- Everybody who buys a ticket receives a prize. Therefore, if there are no prizes, then nobody buys a ticket.

  Let $B(x,y)$: x buys y $T(x)$: x is a ticket
  $P(x)$: x is a prize $R(x,y)$: x receives y

  So we must show
  $$\begin{array}{l} (\forall x)\big((\exists y)(B(x,y) \wedge T(y)) \to (\exists y)(P(y) \wedge R(x,y))\big) \\ \hline \neg(\exists x)P(x) \to (\forall x)(\forall y)(B(x,y) \to \neg T(y)) \end{array}$$

  (1) $(\forall x)\big((\exists y)(B(x,y) \wedge T(y)) \to (\exists y)(P(y) \wedge R(x,y))\big)$ P

  we're going to use CP

  (2) $\neg(\exists x)P(x)$ P
  (3) $(\forall x)\neg P(x)$ t and (2)
  (4) $\neg P(y)$ ui and (3)
  (5) $\neg P(y) \vee \neg R(x,y)$ t and (4)
  (6) $(\forall y)(\neg P(y) \vee \neg R(x,y))$ ug and (5)
  (7) $\neg(\exists y)(P(y) \wedge R(x,y))$ t and (6)
  (8) $(\exists y)(B(x,y) \wedge T(y)) \to (\exists y)(P(y) \wedge R(x,y))$ ui and (1)
  (9) $\neg(\exists y)(B(x,y) \wedge T(y))$ t (7, 8)
  (10) $(\forall y)(\neg B(x,y) \vee \neg T(y))$ t (9)
  (11) $(\forall y)(B(x,y) \to \neg T(y))$ t (10)
  (12) $(\forall x)(\forall y)(B(x,y) \to \neg T(y))$ ug (11)
  (13) $\neg(\exists x)P(x) \to (\forall x)(\forall y)(B(x,y) \to \neg T(y))$ cp (2, 12)