Remarks

Notice that if a variable is free in a formula, then it really does behave like a 'variable' in the colloquial mathematical sense since the formula is effectively a statement about that variable. Hence we could define a "statement" to be a formula having no free variables.

We can give two further deductive rules relevant to predicate calculus, namely

rule ui $\vDash (\forall x) P(x) \longrightarrow P(a)$ for "a" any value in the domain of the variable x.

this rule is commonly called universal instantiation or universal specification.

rule ug $\vDash P(x) \longrightarrow (\forall x) P(x)$ where $P(x)$ is an earlier formula in the sequence of deductions such that x is not a variable having a free occurence in <u>any</u> premise. This rule is universal generalisation, and is typified <u>by</u> proofs starting with "Let x be ..." and then observing some property $P(x)$, so deducing from the initial arbitrariness of x that $P(x)$ holds for all x.

There are, of course, the related rules ...

rule ei $\vDash (\exists x) P(x) \longrightarrow P(a)$ for "a" a 'chosen' value for x lying in the domain of x, even though its actual value is unknown to us.

rule eg $\vDash P(a) \longrightarrow (\exists x) P(x)$, i.e. knowing that there is a value in the domain of x for which $P(x)$ holds allows us to deduce $(\exists x) P(x)$!!