Definitions

A _variable_ is denoted by a letter. (This cryptic definition is to allow a letter, typically from the latter end of the alphabet, to stand intuitively for anything at least locally unknown or undetermined.)

A _constant_, typically denoted by a letter from the front end of the alphabet, denotes a specific well-defined object. For example, a constant could be the number 5, or the number π, or a specific person or country.

A _term_ is a variable or a constant.

An _n-place predicate_ $P(x_1, \ldots, x_n)$ is an expression involving the variables $x_1, \ldots, x_n$ such that assigning appropriate values to these x_i makes P into a statement. For convenience we define a 0-place predicate to be a statement.

A _prime formula_ is an expression obtained from a predicate by substituting (not necessarily distinct) variables for the original variables in P. So for example, if $P(x, y, z)$ is a predicate then $P(x, y, z)$, $P(z, y, x)$, $P(u, v, w)$, $P(x, x, y)$ and $P(z, z, z)$ are examples of prime formulae.

The phrase "for all x", denoted $\forall x$, is a _universal quantifier_, and the phrase "there exists x", denoted $\exists x$, is called an _existential quantifier_.

A _formula_ is an expression built up from prime formulae by using the operators $\neg, \wedge, \vee, \rightarrow$ and $\leftrightarrow$. We also allow the word "formula" to include expressions built from $(\forall x) P$ and $(\exists x) P$, where P is a formula and x is a variable.

To save unnecessary use of parentheses we'll adopt the convention that operators and quantifiers will have the least possible scope, so for example $\exists x A \vee B$ will mean $((\exists x) A) \vee B$.