Noticing that if we show $\models P_1, \ldots, P_m, Q \to R$ then we have effectively shown $P_1, \ldots, P_m \models Q \to R$ allows us to be a little more creative in our derivations. As an example we'll give a different derivation of the previous example, calling this rule cp ...

__Example__

Show that $A \vee B,\ A \to C,\ B \to D \models C \vee D$.

First notice that by tautology #31 $C \vee D$ is equivalent to $\neg C \to D$, so we'll try to show that

$$A \vee B,\ A \to C,\ B \to D,\ \neg C \models D$$

(1)	P_1: $A \vee B$	rule p		
(2)	P_2: $A \to C$	rule p		
(3)	P_3: $B \to D$	rule p		
(4)	P_4: $\neg C$	rule p		
(2,4)	P_5: $\neg A$	rule t	and taut 2	$\models P_2, P_4 \to P_5$
(1,2,4)	P_6: B	rule t	and taut 3	$\models P_1, P_5 \to P_6$
(1,2,3,4)	P_7: D	rule t	and taut 1	$\models P_3, P_6 \to P_7$
(1,2,3)	P_8: $\neg C \to D$	rule cp		
(1,2,3)	P_9: $C \vee D$	rule t	and taut 31	

This approach is sometimes called a "conditional proof". It should be said that many authors prefer to refer to stating a premise (our rule p) as needing no rule, and then list rules of deduction as

rule mp — modus ponens
rule cp — conditional proof (just as we've done)
rule mt — modus tollens

We now move on to the other common first order logic which relates terms, predicates and quantifiers; called __predicate calculus__.