CS 280 Fall 2003

Homework #1
Due: Monday, September 8, in class
If you turn in hand-written work, it must be legible, or it will not be graded.

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1. Defining symmetric difference of sets by $A + B := (A - B) \cup (B - A)$, show that $A + B = \text{Ø} \Leftrightarrow A = B$. (Notice that this means that any equation involving sets can be converted to one whose RHS is Ø.)

2. Show that any set equation involving one unknown set X whose RHS is Ø can be rewritten as $(A \cap X) \cup (B \cap \overline{X}) = \text{Ø}$ where neither A nor B involve X. (Assume the equation is written using only $\cap$, $\cup$ and complement.)

3. Show that $A = B = \text{Ø}$ iff $A \cup B = \text{Ø}$. (Notice that this means that the equation in part 2 is equivalent to two simultaneous equations $A \cap X = \text{Ø}$ and $B \cap \overline{X} = \text{Ø}$.

4. Show that the pair of simultaneous equations in part 3 has a solution iff $B \subseteq \overline{A}$. Moreover, any X with $B \subseteq X \subseteq \overline{A}$ is such a solution.

5. Apply this to find necessary and sufficient conditions that the equation $X \cup C = D$ has a solution.

6. In the set $Z_{>0}{}^{*} \times Z_{>0}{}^{*}$, define $(a,b) \sim (c,d)$ iff $a + d = b + c$. Prove that ~ is an equivalence relation and describe the equivalence classes.

7. (Question 12 on page 20.)

8. Suppose that $f : A \to B$ is an "onto" function and define a relation ~ on A by $a \sim a'$ iff $f(a) = f(a')$. Show that ~ is an equivalence relation. If we let $C = A / \sim$ and $\eta : A \to C$ by $\eta(a) := [a]$, show that there exists a "one-to-one and onto" function $g : C \to B$ such that $f = g_0\eta$.

9. Define "<" on $Z_{>0}{}^{*}$ by $p < q$ iff $p \mid q$. Show that < is a partial order.

* It was not possible to render this character correctly – please substitute the Z with the character "zed".

Algebra of Sets

1. $A \cup (B \cup C) = (A \cup B) \cup C$

2. $A \cup B = B \cup A$

3. $A \cup (B \wedge C) = (A \cup B) \cap (A \cup C)$

4. $A \cup \emptyset = A$

5. $A \cup \overline{A} = U$

6. $A \cup B = A \quad \forall B \Rightarrow B = \emptyset$

7. $(A \cup B = U) \wedge (A \cap B = \emptyset) \Rightarrow B = \overline{A}$

8. $\overline{\overline{A}} = A$

9. $\overline{\emptyset} = U$

10. $A \cup A = A$

11. $A \cup U = U$

12. $A \cup (A \cap B) = A$

13. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (de Morgan)

Duality

Swap $\cup$ $\cap$
and $\emptyset$ u

Then using $\cup$, $\cap$, and complement, dual theorem true.