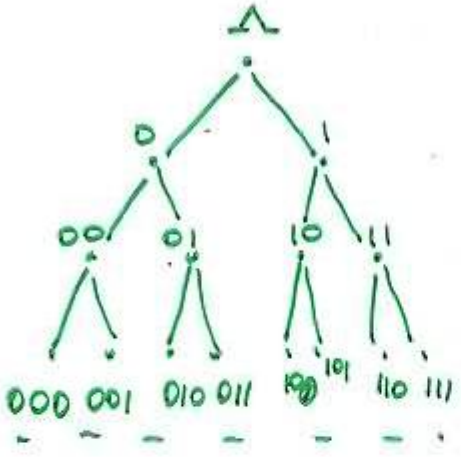
LEC 36 11/29/99

TREE TRAVERSAL

LABELING VERTICES IN AN ORDERED ROOTED TREE



SET OF BINARY STRINGS

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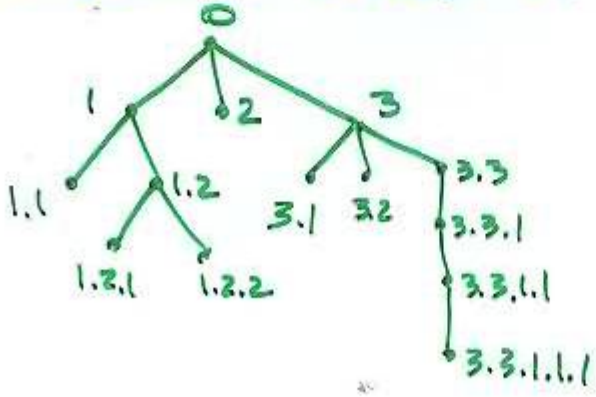
BINARY ORDERED ROOTED TREE + ORDERING OF CHILDREN

UNIVERSAL ADDRESS SYSTEM: ROOT IS LABELED 0 AND ITS CHILDREN AT LEVEL 1 WITH 1, 2, ..., k_0. FOR EACH VERTEX AT LEVEL $n \geq 1$ WITH LABEL A ITS CHILDREN GET LABELS A.1 A.2 ... A.n_v



CAN BE TOTALLY ORDERED LEXICOGRAPHICALLY

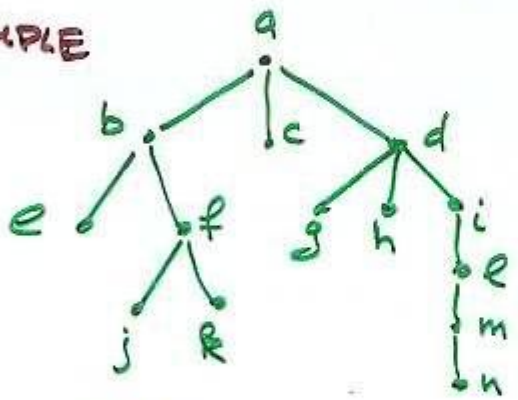
TRAVERSING TREES: PREORDER
ALL DEFINED INORDER
RECURSIVELY POSTORDER TRAVERSALS

IN ALL DEFINITIONS BELOW T IS AN ORDERED ROOTED TREE WITH ROOT r; $T_1 \dots T_n$ - SUBTREES AT r

DEF. 1. **PREORDER TRAVERSAL** OF T $PrT(T)$

$PrT(T) = r, PrT(T_1), \dots, PrT(T_n)$

EXAMPLE



$PrT(T) = a, b, e, f, j, k, c, d, g, h, i, l, m, n.$

DEF. 2. **INORDER TRAVERSAL** $InT(T)$

$InT(T) = InT(T_1), r, InT(T_2), \dots, InT(T_n)$

EXAMPLE: $InT(T) = e, b, j, f, k, a, c, g, d, h, l, i, m, n$

DEF. 3 **POSTORDER TRAVERSAL** $PoT(T)$

$PoT(T) = PoT(T_1), \dots, PoT(T_n), r$

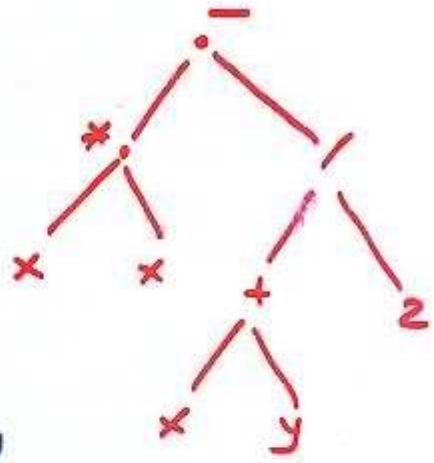
EXAMPLE: $PoT(T) = e, j, k, f, b, c, g, h, l, m, n, i, d, a.$

TERM NOTATIONS

TERM = LABELED TREE

$$x^2 - \frac{x+y}{2} =$$

$$= (x * x) - [(x+y)/2]$$



DEF. PREFIX FORM OF AN EXPRESSION = PREORDER TRAVERSAL = POLISH NOTATION

EXAMPLE: PREFIX FORM OF $x^2 - \frac{x+y}{2} = -*xx/+xy2$

UNAMBIGUOUS!

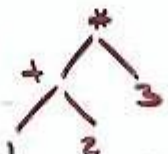
DEF. POSTFIX FORM = = POSTORDER TRAVERSAL = = REVERSE POLISH NOTATION

POSTFIX FORM OF $x^2 - \frac{x+y}{2}$ = $xx*xy+2/-$

DEF. INFIX FORM = $x*x - x+y/2$ = INORDER TRAVERSAL + FULL PARENTHESING!

OTHERWISE AMBIGUOUS

EXAMPLE: $(1+2)*3$ =



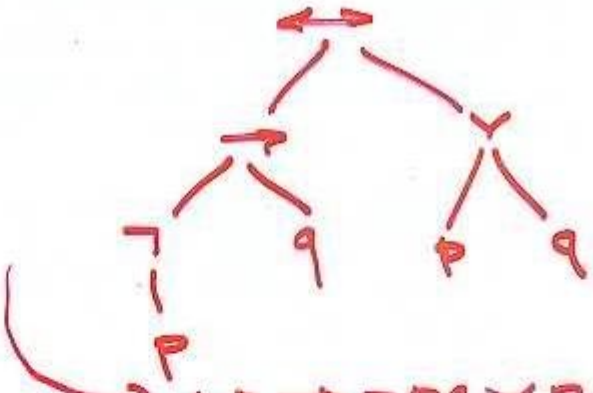
$\longrightarrow 1+2*3$

$1+(2*3) =$

$\longrightarrow 1+2*3$

EXAMPLE $(\neg p \to q) \leftrightarrow (p \vee q)$

ROOTED TREE FORM



PREFIX FORM

$\leftrightarrow \to \neg p q \vee p q$

POSTFIX FORM

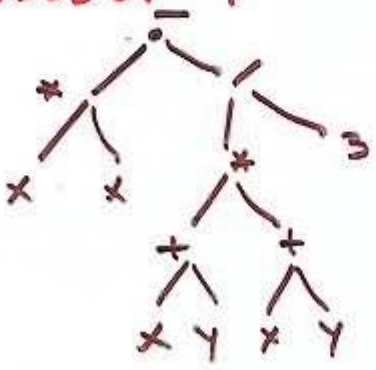
$p \neg q \to p q \vee \leftrightarrow$

INFIX FORM

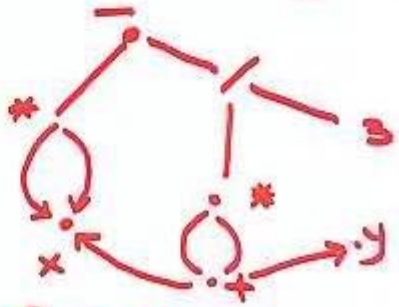
$(p \neg \to \neg q) \leftrightarrow (p \vee q)$

DAG FORM = DIRECTED ACYCLIC GRAPHS = EACH SUBTREE IS STORED ONLY ONCE. EXAMPLE $x^2 - \frac{(x+y)^2}{3}$

CAN GIVE AN EXPONENTIAL SPEED UP !



13 VERTICES



8 VERTICES