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lec. 35. Nov. 22.Tree: connected acyclic undirected graph.Thm 1. (p. 529.) A graph G is a tree iff there is a unique path between any two vertices.

- Prf.
- Suppose G is a tree. Let u, v be any two vertices in G . G connected $\Rightarrow \exists$ a path p_1 between u and v . Suppose $\exists$ also a path $p_2 \neq p_1$, from u to v . Then $p_1 + p_2$ form a cycle, contradicting G being a tree. Thus p_2 cannot exist, and p_1 is unique.
 - Now suppose G is a graph with a unique path between any u, v . Then G is connected by definition. Suppose G has a cycle and let x, y be on the cycle. Then there are two different paths from x to y (think!) — contradiction. So G has no cycles, and it is a tree.

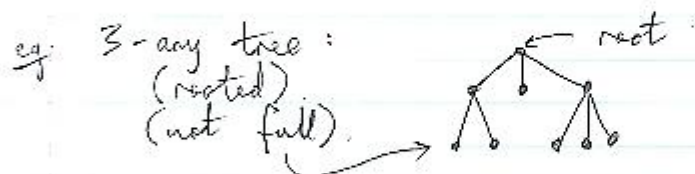
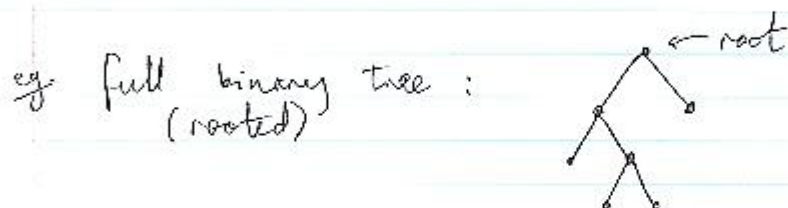
Def. rooted tree: just pick any vertex as the root r , and let the rest of the tree "hang" from r . Can now speak of parents and children.

Def. Leaf: a tree vertex w/ no children.
Internal node: a tree vertex w/ children.

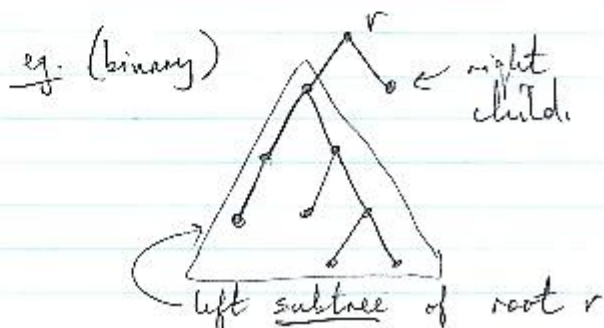
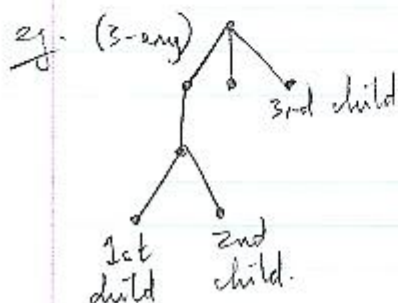
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Def. (p. 531).

- A tree is m-ary if every vertex has at most m children.
- full m-ary tree: everyone (except leaves) has exactly m children.



Def. Ordered tree: order of children matters.
So, can speak of left and right children (binary trees), or 2nd or 6th child (general m-ary tree).

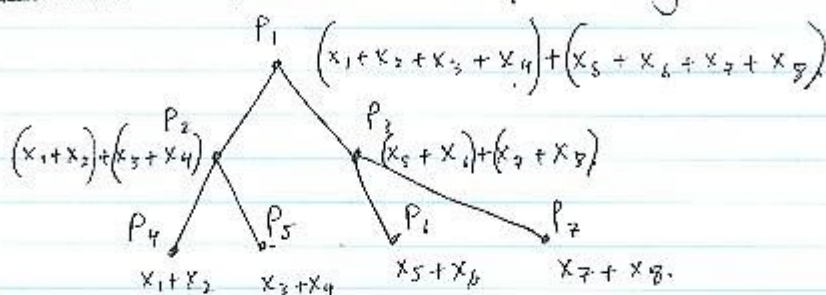


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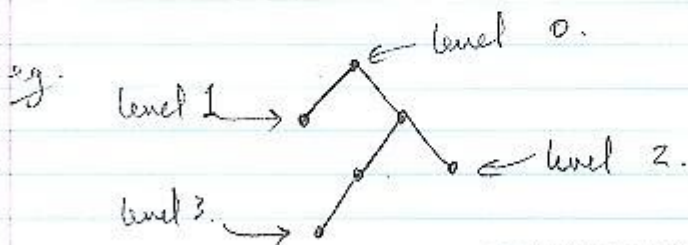
Example 7, p. 535 File system (eg. UNIX, DOS).

Example 8, p. 535 Parallel processing.



Def. The level of a vertex v in a tree T is the length of the path from the root r to v (ie. # of edges from r to v).

• Height of a tree: maximum level of all vertices.



TREE WITH HEIGHT 3.

Binary Search Trees: Example 1, p. 541-542.

Decision Trees: Example 2, p. 544.

Prefix trees: Try to save space. High-frequency letters $\rightarrow$ shorter bit strings. Prefix code: bit string for x must not be the first part of bit string for y .
Figure 4.