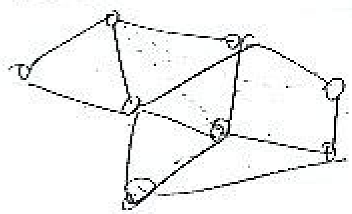
Def. $\boxed{\text{Planar graph: can be drawn in the plane w/out any edges crossing.}}$ SAY IT!

eg.



PLANAR REPRSENTATION:

~~eg. nonplanar K_5~~

eg. nonplanar $K_{3,3}$. (proof in book)

eg. planar K_4.

Euler's ~~Theorem~~ Formula.

G connected, planar $\Rightarrow r = e - v + 2$.

pf: induction on subgraphs $G_1, G_2, \ldots, G_e$.

BASIS. G_1. ✓

IND: $G_n \to G_{n+1}$.

Two cases: ① New edge touches no new vertices
② " " " a new vertex.

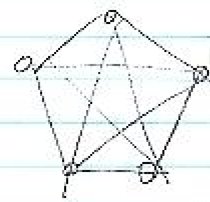
QED: $(G_e = G)$.

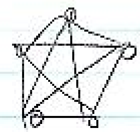
eg. Demonstrate on my eg. and K_4

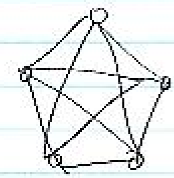
Corollary (p. 505).

G connected, planar $\Rightarrow$ $e \leq 3v-6$.

e.g. K_5 nonplanar.

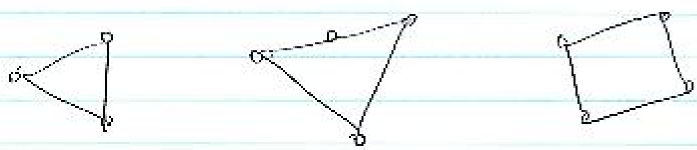






Graph Homeomorphisms.

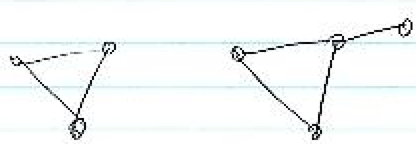
e.g.



All cycles C_n are homeomorphic to each other.

This is *weaker* than *isomorphism*.

e.g.



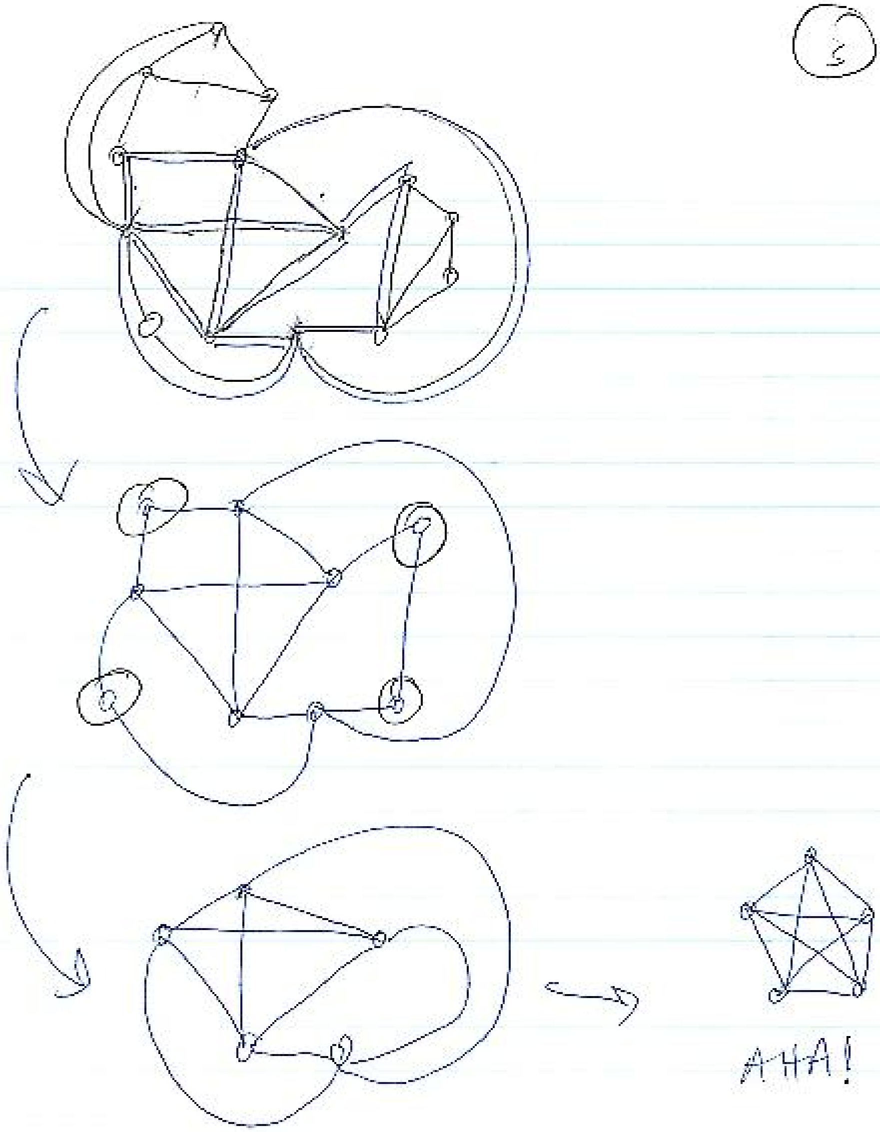
not homeomorphic.

Kuratowski's Theorem.

Graph *nonplanar* $\Longleftrightarrow$ has subgraph homeo to K_5 *OR* has subgraph homeo to $K_{3,3}$.

Application: VLSI.

eg.



<u>Summary.</u>

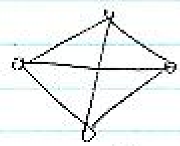
- Planar graphs.
- Euler $r = e - v + 2$.
- (Connected + planar) $\Longrightarrow$ $e \leq 3v - 6$.
- Kuratowski's Theorem.

Graph Coloring.

<u>Coloring:</u> adjacent vertices have <u>different</u> colors.

e.g. n vertices $\rightarrow$ n colors.
OK, but too easy.

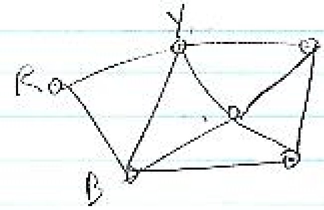
e.g.



K_n: <u>must</u> have n colors.

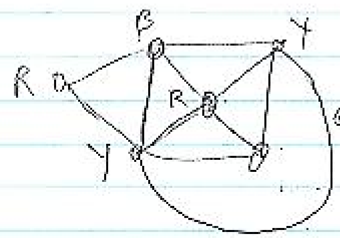
chromatic number $\Leftarrow$ SAY IT!

e.g.



$\chi(G) = 3$.

e.g.



Two greens adjacent!
!

$\chi(G) > 3$.

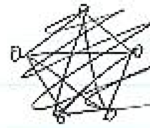
actually $\chi(G) = 4$.

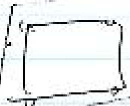
Actually... a famous Theorem:

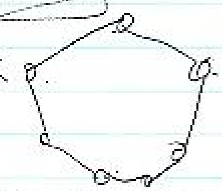
Four-Color Theorem.

$$G \text{ planar} \Rightarrow \chi(G) \leq 4!$$

- proved by Appel & Haken (1976)
- first posed in 1850; many wrong proofs
- partly done by computer! (~2000 ~~[illegible]~~ cases checked)

eg.  ~~K_5 needs 5 colors~~

eg. Even cycles have $\chi = 2$ 
~~are 2 colorable~~

Odd cycles have $\chi = 3$. 

Coincidence: bipartite $\iff$ 2-colorable.

Finding $\chi(G)$ is NP-complete

- exponential (worst-case) time
- interesting fact: even getting a $2\chi(G)$-coloring is NP-complete.

Eg. 6 p. 516.

- Channels 2-13 need 150 miles separation.
- Easy: make a graph: (1) each station is a vertex. (2) Edge $\iff$ 2 stations are within 150 miles.

Eg. 7, p. 517. Compilers.

Loop: how many vars. in loop $\Rightarrow$ CPU registers
For each var. $\Rightarrow$ vertex.
Edge $\iff$ 2 vars. need registers during same loop.

For a given loop.