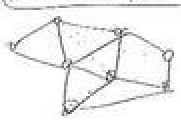
1

Def. Planar graph: can be drawn in the plane without any edges crossing.

SAY IT!

eg.



PLANAR REPRESENTATION.

~~eg. nonplanar K_5~~

eg. nonplanar $K_{3,3}$. (proof in book)

eg. planar K_4.

Euler's ~~Theorem~~ Formula.

G connected, planar $\Rightarrow r = e - v + 2$.

pf. induction on subgraphs $G_1, G_2, \ldots, G_e$.
BASIS G_1. ✓

IND. $G_n \rightarrow G_{n+1}$.
Two cases: (1) New edge touches no new vertices
(2) New edge touches a new vertex.

QED. ($G_e = G$).

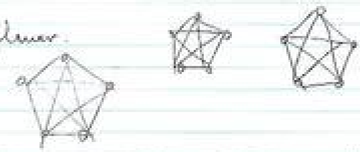
eg. Demonstrate on my eg. and K_4

2

Corollary (p. 505).

G connected, planar $\Rightarrow e \leq 3v - 6$.

eg. K_5 nonplanar.



Graph Homeomorphisms.

eg.

All cycles C_n are homeomorphic to each other.

This is weaker than isomorphism.

eg.

not homeomorphic.

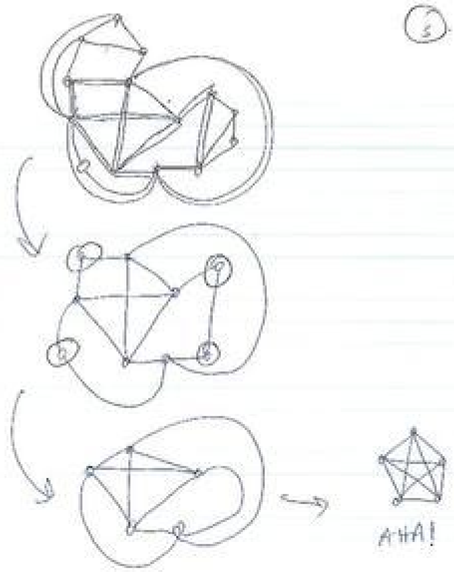
Kuratowski's Theorem.

Graph nonplanar $\Leftrightarrow$ has subgraph homeo to K_5 OR has subgraph homeo to $K_{3,3}$.

Application: VLSI.

3

eg.



AHA!

Summary.
- Planar graphs.
- Euler $r = e - v + 2$.
- (Connected + planar) $\Rightarrow e \leq 3v - 6$.
- Kuratowski's Theorem.

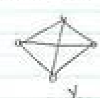
4

Graph Coloring.

Coloring: adjacent vertices have different colors

eg. n vertices $\rightarrow$ n colors.
OK, but too easy.

eg.

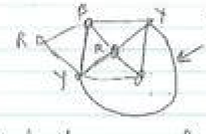


K_n: must have n colors.

$\chi(G) = 3$ $\boxed{\text{chromatic number}}$ ← SAY IT!

eg.

eg.



Two greens adjacent!

$\chi(G) > 3$.

actually $\chi(G) = 4$.

Actually... a famous Theorem.

$\boxed{\text{Four-Color Theorem. } G \text{ planar} \Rightarrow \chi(G) \leq 4!}$

- proved by Appel & Haken (1976)
- first posed in 1850; many wrong proofs
- proof done by computer! (1500 cases checked)

eg.  ~~K_5 needs 5 colors~~

eg. Even cycles have $\chi = 2$ 
~~are 2-colorable~~

Odd cycles have $\chi = 3$. 

$\boxed{\text{Coincidence: bipartite} \Leftrightarrow \text{2-colorable.}}$

$\boxed{\text{Finding } \chi(G) \text{ is NP-complete}}$

- exponential (worst-case) time
- interesting fact: even getting a $2\chi(G)$-coloring is NP-complete.

Eg. 6 p. 516.

- channels 2-13 need 150 miles separation.
- Easy: make a graph ① each station is a vertex
  ② Edge ~~and~~ 2 stations are within 150 miles.

Eg. 7, p. 517. Compilers.

Loop: how many vars in loop $\Rightarrow$ CPU registers

For each var. $\Rightarrow$ vertex.

Edge $\Leftrightarrow$ 2 vars. need register during same loop.

For a given loop.