LEC. 33 11/17/99

SHORTEST PATHS. WEIGHTED GRAPH = A NUMBER ASSIGNED TO EACH EDGE

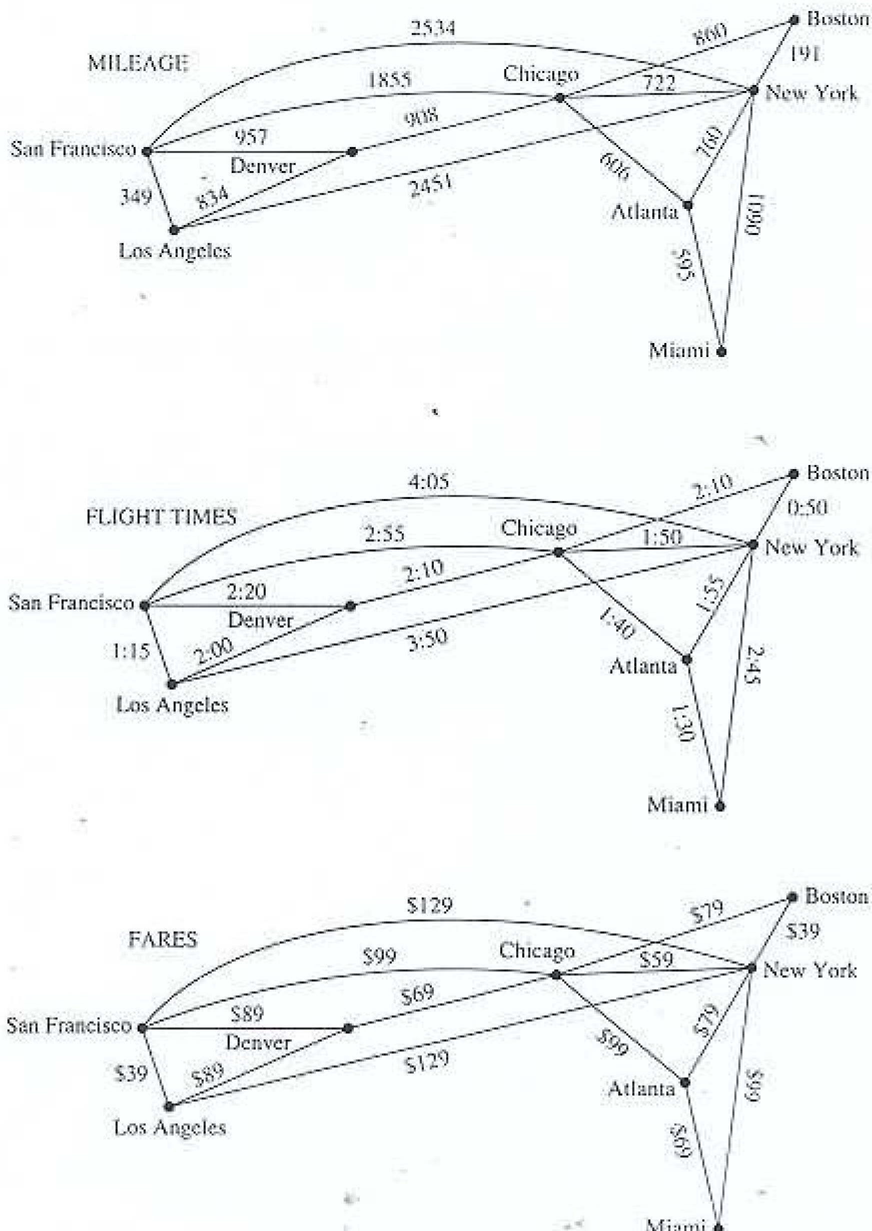


FIGURE 1 Weighted Graphs Modeling an Airline System.

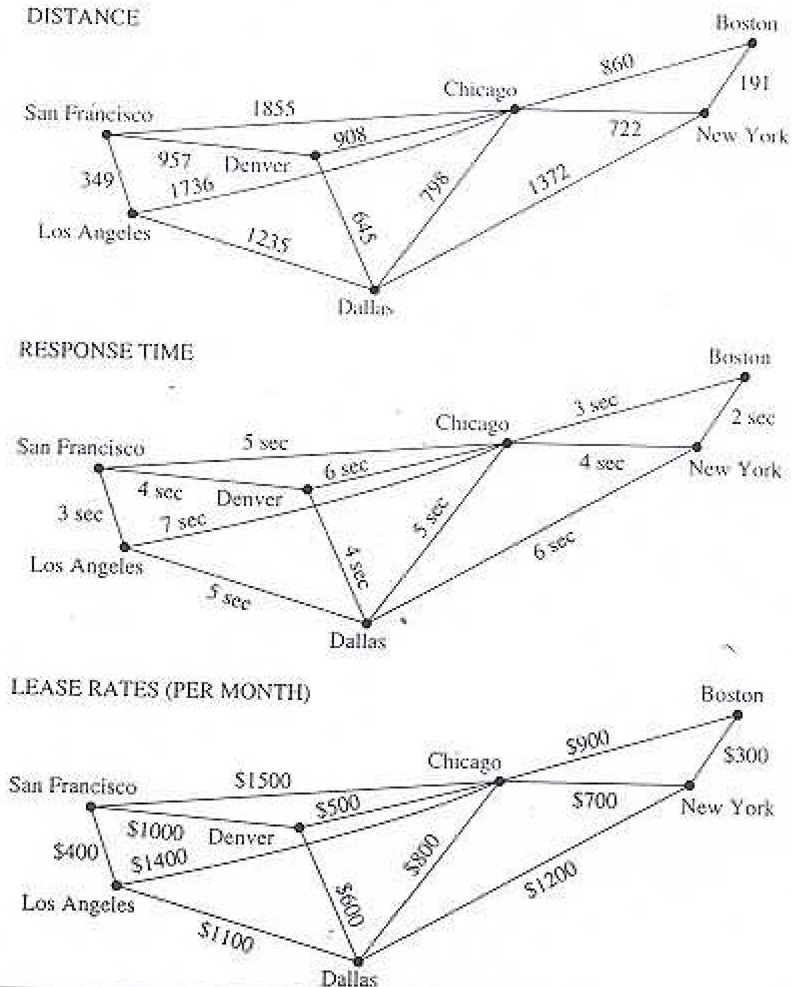
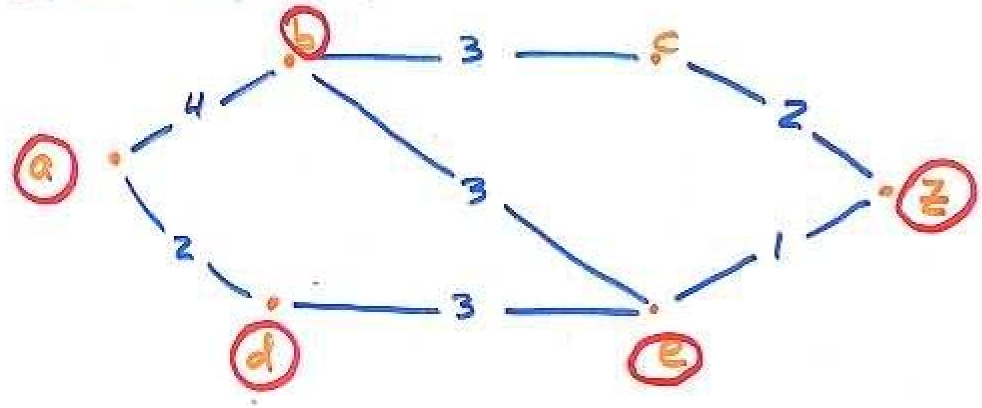


FIGURE 2 **Weighted Graphs Modeling a Computer Network.**

THE LENGTH OF A PATH IN A WEIGHTED GRAPH =
= THE SUM OF THE WEIGHTS OF THE EDGES OF THIS PATH.

WHAT IS THE SHORTEST PATH BETWEEN TWO GIVEN VERTICES?



FROM a TO z? DIJKSTRA ALGORITHM:

FIND THE LENGTH OF THE SHORTEST PATH FROM a TO SUCCESSIVE VERTICES, UNTIL z IS REACHED

$\emptyset = S_0$

a $= S_1$, $L(a)=0$

a, d(2) $= S_2$ $L(d)=2$

a, d(2), b(4) $= S_3$ $L(b)=4$

a, d(2), b(4), e(via d; 5) $= S_4$ $L(e)=5$

a, d(2), b(4), e(via d; 5), z(via d, e; 6) $= S_5$, $L(z)=6$

ALGORITHM 1 Dijkstra's Algorithm.

procedure *Dijkstra*(G: weighted connected simple graph, with
  all weights positive)
{G has vertices $a = v_0, v_1, \ldots, v_n = z$ and weights $w(v_i, v_j)$
  where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in G}
for $i := 1$ **to** n
  $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is zero and all
  other labels are ∞, and S is the empty set}
while $z \notin S$
begin
  $u :=$ a vertex not in S with $L(u)$ minimal
  $S := S \cup \{u\}$
  for all vertices v not in S
    if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
  {this adds a vertex to S with minimal label and updates the
    labels of vertices not in S}
end {$L(z) =$ length of shortest path from a to z}

$$L_k(u) = \begin{cases} \text{THE SHORTEST DISTANCE BETWEEN } a \text{ AND } u \text{ IN } S_k \\ \infty, \text{ O.W.} \end{cases}$$

$S_{k+1} = S_k \cup \{u\}$ WHERE $u \in S_k$ WITH THE MINIMAL LABEL

Th. DIJKSTRA ALGORITHM FINDS THE LENGTH OF A SHORTEST PATH IN A CONNECTED SIMPLE UNDIRECTED WEIGHTED GRAPH. $O(n^2)$ ADDITION AND COMPARISONS.

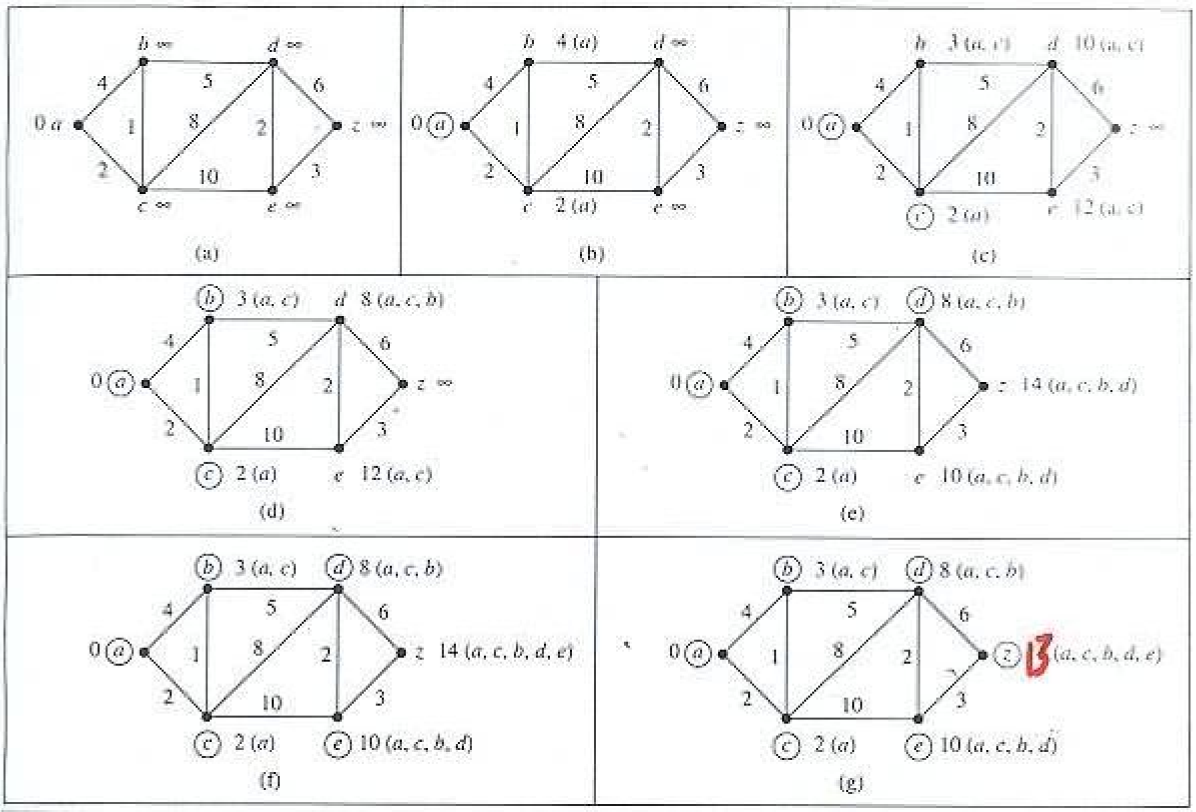


FIGURE 4 Using Dijkstra's Algorithm to Find the Shortest Path from a to z.

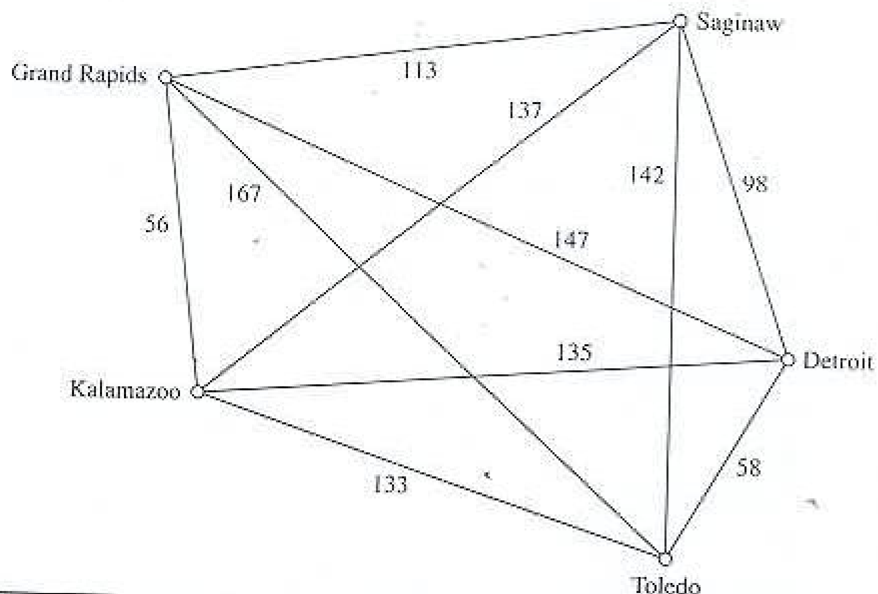


FIGURE 5 The Graph Showing the Distances Between Five Cities.

THE TRAVELING SALESMAN PROBLEM =
A HAMILTON CIRCUIT WITH MINIMUM TOTAL WEIGHT
NP-COMPLETE PROBLEM