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## CONNECTIVITY

A PATH OF LENGTH $n$ FROM $u$ TO $v$

IN A SIMPLE GRAPH: $u = x_0, x_1, \ldots, x_n = v$ SUCH THAT $\{x_i, x_{i+1}\} \in E$

IN AN ARBITRARY UNDIRECTED GRAPH: $e_1, \ldots, e_n$ SUCH THAT $f(e_i) = \{x_{i-1}, x_i\}$, $f(e_{i+1}) = \{x_i, x_{i+1}\}$, $x_0 = u$, $x_n = v$

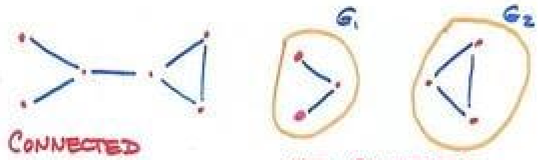
IN A DIRECTED GRAPH: $f(e_i) = (x_{i-1}, x_i)$, $f(e_{i+1}) = (x_i, x_{i+1})$

CIRCUIT (CYCLE): $u = v$

SIMPLE PATH: DOES NOT CONTAIN THE SAME EDGE MORE THAN ONCE

GRAPH IS CONNECTED: THERE IS A PATH BETWEEN EVERY PAIR OF DISTINCT VERTICES



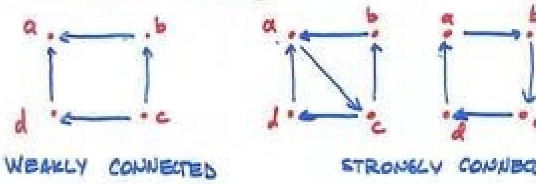



CONNECTED

NOT CONNECTED

$G_1, G_2$ - CONNECTED COMPONENTS

DIRECTED GRAPHS




WEAKLY CONNECTED (NO DIRECTED PATH FROM a TO c)

STRONGLY CONNECTED

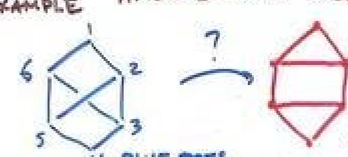
ISOMORPHIC INVARIANTS: CONNECTIVITY, EXISTENCE OF A SIMPLE CIRCUIT OF A PARTICULAR LENGTH, ETC.

HELP TO FIND ISOMORPHISMS OR TO ESTABLISH THAT THERE IS NO ONE.



EXAMPLE

AFTER SEVERAL FAILED ATTEMPTS TO BUILD AN ISOMORPHISM WE FIND A INVARIANT THAT DISTINGUISHES THESE GRAPHS.




BLUE DOES NOT!

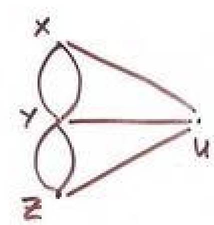
RED HAS A SIMPLE CIRCUIT OF LENGTH 3

**def** EULER CIRCUIT IN A GRAPH G ≡ SIMPLE CIRCUIT CONTAINING EVERY EDGE OF G.


SEVEN BRIDGES OF KÖNIGSBERG

TRAVEL ACROSS ALL THE BRIDGES WITHOUT CROSSING ANY BRIDGE TWICE, AND RETURN TO THE STARTING POINT






SEVEN BRIDGES GRAPH.

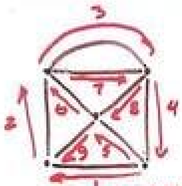
Th (EULER) A CONNECTED MULTIGRAPH HAS AN EULER CIRCUIT IFF EACH OF ITS VERTICES HAS EVEN DEGREE.

EXAMPLES.

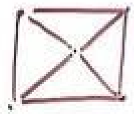
NO EULER CIRCUITS

THERE ARE EULER CIRCUITS

COROLLARY: SEVEN BRIDGES GRAPH DOES NOT HAVE AN EULER CIRCUIT *def* EULER PATH — SIMPLE PATH CONTAINING EVERY EDGE OF G



EULER PATH



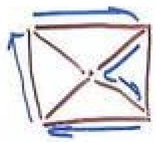
NO EULER PATHS !

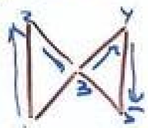
*th.* (EULER) A CONNECTED MULTIGRAPH G HAS AN EULER PATH BUT NOT A EULER CIRCUIT IFF G HAS EXACTLY TWO VERTICES OF ODD DEGREE

HAMILTON PATH : CONTAINS EVERY VERTEX EXACTLY ONCE

HAMILTON CIRCUIT : HAMILTON PATH WHICH IS A CIRCLE.

HAMILTON CIRCUIT!



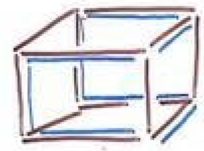


HAMILTON PATH

NO HAMILTON PATHS



EXAMPLE. HAMILTON CIRCUIT FOR $Q_3$



*th.* SUFFICIENT CONDITION FOR A HAMILTON CIRCUIT: EACH VERTEX HAS THE DEGREE AT LEAST $n/2$, $n = |V|$, $n \geq 3$.

(ENOUGH EDGES)

NO. SIMPLE CRITERIA IS KNOWN.

NP-COMPLETE PROBLEM