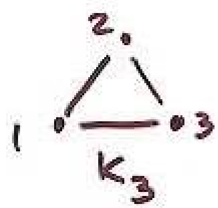
HW 7.3: 14, 16, 40, 42, 58, 62 LEC. 31 11/12/99

ADJACENCY MATRIX $G = (V, E)$ - SIMPLE GRAPH

$A_G = [a_{ij}] \quad 1 \le i, j \le |V|$

$$a_{ij} = \begin{cases} 1, & \text{if } \{v_i v_j\} \in E \\ 0, & \text{o.w.} \end{cases}$$

EXAMPLE




$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

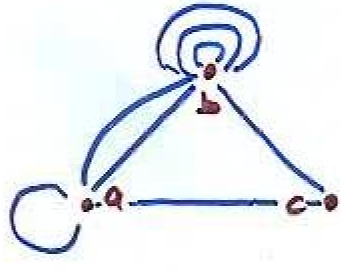
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$


$a_{ii} = 0$ SINCE G HAS NO LOOPS

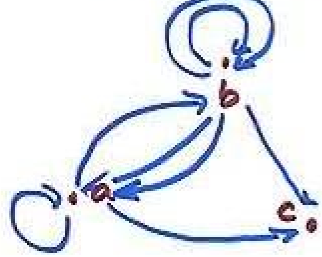
$a_{ij} = a_{ji}$ SINCE $\{v_i v_j\} = \{v_j v_i\}$

## MATRICES FOR MULTI-(PSEUDO-)GRAPHS

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$




## MATRECES FOR DIRECTED GRAPHS




$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

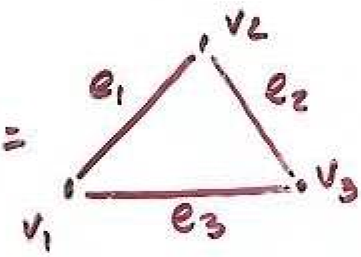
NOT NECESSARILY SYMMETRIC MATRIX

## INCIDENCE MATRICES

$G = (V, E)$

$$M_G = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{array} \begin{array}{c} e_1 \; e_2 \; \cdots \; e_m \\ \left[ \begin{array}{ccc} & & \\ & m_{ij} & \\ & & \end{array} \right] \end{array}$$

$$m_{ij} = \begin{cases} 1, & \text{IF } e_j \text{ IS INCIDENT WITH } v_i \\ 0, & \text{O.W.} \end{cases}$$

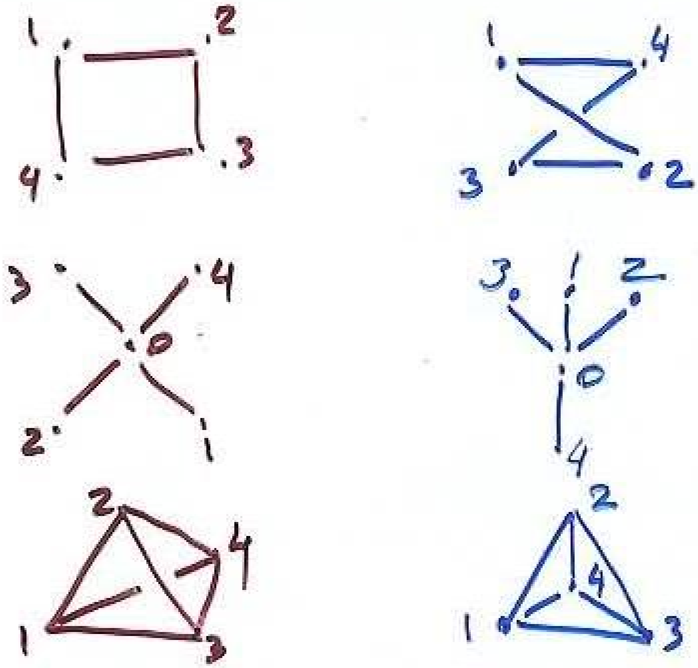
$G =$ 


$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = M$$

TWO ONES IN EACH COLUMN

ALSO WORKS FOR ALL OTHER SORTS OF GRAPHS

## ISOMORPHISM OF GRAPHS

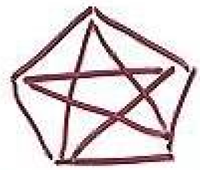



IDENTICAL (ISOMORPHIC) GRAPHS

**def.** $G_1 = (V_1 E_1)$ ; $G_2 = (V_2 E_2)$ SIMPLE GRAPHS ARE ISOMORPHIC IF THERE IS a bijection $f : V_1 \to V_2$ such that $\{a, b\} \in E_1 \iff \{f(a), f(b)\} \in E_2$.

INVARIANTS OF ISOMORPHISMS

- # OF VERTICES
- # OF EDGES
- DEGREES OF VERTICES
- CONNECTIVITY
- etc.

EXAMPLE. $K_5$  $W_4$ 

NOT ISOMORPHIC: 10 EDGES IN $K_5$ vs.
8 EDGES IN $W_4$

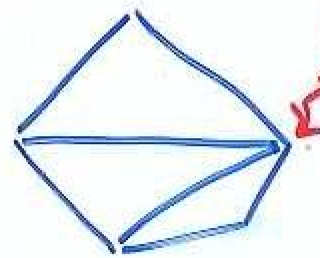
 

NOT ISOMOPRHIC: ONE HAS A VERTEX OF DEGREE 1...

**Th.** A BIJECTION $f: V_1 \longrightarrow V_2$ IS AN ISOMPRPHSM OF $(V_1, E_1)$ AND $(V_2 E_2)$ IFF THE ADJACENCY MATRIX OF $(V_1 E_1)$ COINCIDES WITH THE ADJACENCY MATRIX OF $(f(V_1), E_2)$

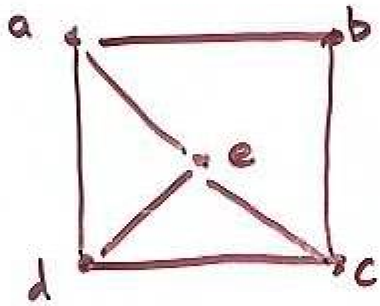
EXAMPLE.

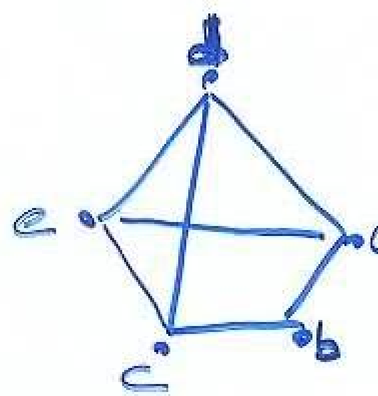
DETERMINE WHETHER THE GIVEN PAIR OF GRAPHS IS ISOMORPHIC.

deg = 4

NO!




$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$