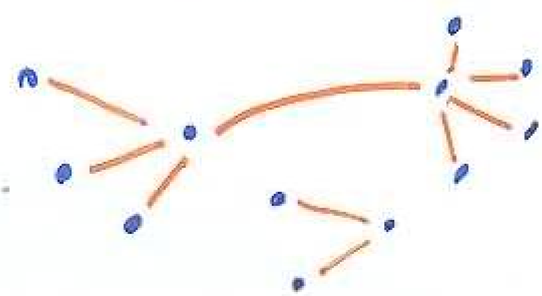
LEC. 29 11/05/99

GRAPHS

$$\text{GRAPH} \approx \underbrace{\text{SET}}_{\text{VERTICES}} + \underbrace{\text{BINARY RELATION}}_{\text{EDGES}}$$

<u>def</u> A <u>SIMPLE GRAPH</u> $G = (V, E)$
$V \neq \emptyset$ SET OF VERTICES
E SET OF UNORDERED PAIRS OF DISTINCT VERTICES

EXAMPLE COMPUTER NET CONNECTED BY TELEPHONE LINES. DATA FLOWS BOTH WAYS. NO COMPUTER IS CONNECTED TO ITSELF BY TELEPHONE



MULTIGRAPH ≈ GRAPH WITH MULTIPLE EDGES

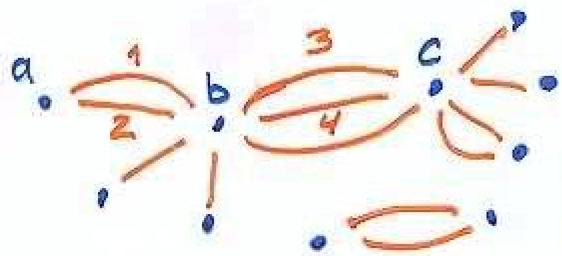
def A <u>MULTIGRAPH</u> $G = (V, E, f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

f FUNCTION FROM E TO THE SET OF UNORDERED PAIRS OF VERTICES

EXAMPLE. COMPUTER NET WITH MULTIPLE TELEPHONE LINES



FUNCTION f READS THE PAIR OF VERTICES CONNECTED BY A GIVEN EDGE

$f(1) = f(2) = \{a, b\}$. $f(3) = f(4) = \{b, c\}$

PARALLEL EDGE'S $f(e_1) = f(e_2)$

STILL NO LOOPS, DIRECTION DOES NOT MATTER

AUTO, AIRPLANE CONNECTION MAPS

PSEUDOGRAPH ≈ MULTIGRAPH WITH LOOPS

def <u>A PSEUDOGRAPH</u> $G = (V, E, f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

f VERTICES READING FUNCTION FROM E TO SET OF UNORDERED PAIRS OF VERTICES NOT NECESSARILY DISTINCT.

EXAMPLE. A COMPUTER NET WITH MULTIPLE TELEPHONE LINES, INCLUDING SELF CONNECTED FOR DIAGNOSTIC PURPOSES.

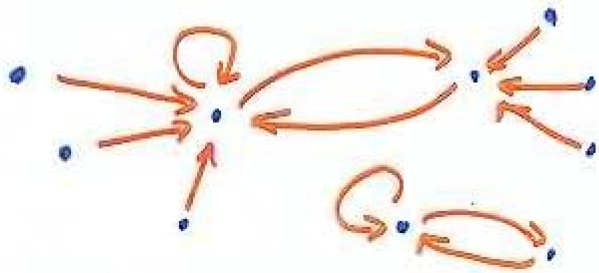
DIRECTED GRAPH ≈ GRAPH WITH DIRECTED EDGES

def. A <u>DIRECTED GRAPH</u> $G = (V, E)$

$V \neq \emptyset$ SET OF VERTICES

$E \subseteq V \times V$. EDGES = ORDERED PAIRS OF VERTICES

EXAMPLE. COMPUTER NET WITH ASSYMMETRIC LINES, NO MULTIPLE LINES IN THE SAME DIRECTION.

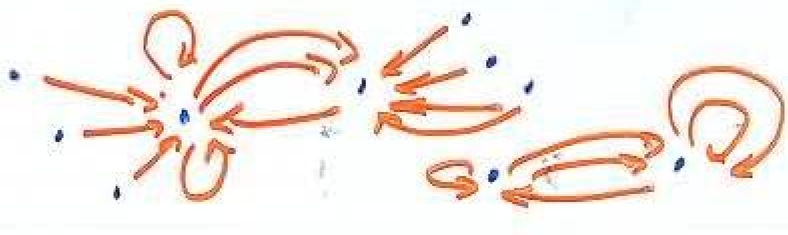


<u>DIRECTED MULTIGRAPH</u> $G = (V, E, f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

$f : E \longrightarrow V^2$ READS INITIAL AND TERMINAL VERTICES OF A GIVEN EDGE

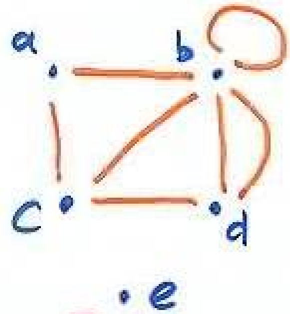


def <u>ADJACENT VERTICES</u> $u, v \in V$ SUCH THAT $\{u,v\} \in E$. $e = \{u,v\}$ <u>CONNECTS</u> u, v. e <u>IS INCIDENT</u> WITH u, v. u, v ARE <u>ENDPOINTS</u> OF e.

def <u>DEGREE OF</u> A VERTEX = # OF EDGES INCIDENT WITH IT

(VALID FOR ALL PSEUDOGRAPHS)

(A LOOP CONTRIBUTES TWICE!)



AN ISOLATED VERTEX

deg (a) = 2

deg (b) = 6

deg (c) = 3

deg (d) = 3

deg (e) = 0

Th. (THE HANDSHAKING THEOREM) $G = (V, E)$ PSEUDOGRAPH (UNDIRECTED)

$e = |E|$

$$2e = \sum_{v \in V} \deg(v)$$

<u>PROOF</u>. EVERY EDGE IS COUNTED TWICE IN Σ.

Th. AN UNDIRECTED (PSEUDO)GRAPH HAS AN EVEN NUMBER OF VERTICES OF ODD DEGREE.

PROOF. LET $V = V_1 + V_2$ WHERE V_1 = VERTICES WITH EVEN DEGREES, V_2 - ODD

$$\underbrace{2e}_{\text{EVEN}} = \sum_{v \in V} deg(v) = \underbrace{\sum_{v \in V_1} deg(v)}_{\text{EVEN}} + \sum_{v \in V_2} deg(v)$$

$$\Rightarrow \sum_{v \in V_2} deg(v) \text{ IS EVEN} \Rightarrow |V_2| \text{ IS EVEN.}$$